Private Equity Indices Based on Secondary Market Transactions*

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June 28, 2018

Abstract

Measuring the performance of private equity investments (buyout and venture) is typically only possible over long horizons because the IRR on a fund is only observable following the fund's final distribution. We propose a new approach to evaluating performance using actual prices paid for funds in secondary markets. We construct indices of buyout and venture capital returns using a proprietary database of secondary market prices between 2006 and 2017. Using this data we find strong evidence that buyout funds outperformed public equity markets on both an absolute and risk adjusted basis over this period. In contrast, venture funds performed about as well as pubic equity markets with alphas that are insignificant from zero. We also find that our transaction-based indices exhibit significantly higher betas and volatilities, and significantly lower Sharpe ratios and correlations with public equity markets relative to NAV-based indices built from Preqin and obtained from Burgiss. There are a number of potential uses for these indices; in particular, they provide a way to track the returns of the buyout and venture capital sectors on a quarter-to-quarter basis and to value illiquid stakes in funds.

JEL classification: G11, G23, G24

Key words: Private Equity, Secondary Market for Private Equity Funds, Market Index

*We are grateful to the partners at an anonymous intermediary for providing us with data. We thank Brigham Frandsen, Jason Gull, Jonathan Jensen, and seminar participants at Texas Tech for useful comments and suggestions. We also thank Greg Adams and Hyeik Kim for excellent research assistance.

1. Introduction

Private equity has become an important asset class for institutional investors. A 2017 survey of institutional investors by consulting firm NEPC finds that 88% are invested in private equity with nearly a third having an allocation greater than 10% (Whyte, (2017)). The vast majority of private equity investors make capital commitments when the funds are initiated and hold them until the final distribution, which is often 12 to 15 years after the initial capital commitment. The return on the fund is determined by the returns on the individual portfolio companies in which the fund invests, and is only fully observable following the fund's final distribution.[1] Therefore, it is difficult for investors to know the value of their private equity portfolio at any point in time, even though the value of the fund's portfolio companies fluctuate with firm-specific and economy-wide news in the same manner as public equities.

The lack of information about private equity funds' values and the way in which they change over time stands in contrast to public stocks, for which there exist active markets where investors trade securities. While active trading markets for investments in private equity funds did not exist prior to 2000, in the early 2000s, a secondary market developed on which limited partners could transact their stakes in private equity funds. In this paper, we use data from this market obtained from a large intermediary in the market to evaluate the fundamentals of the funds themselves in a similar manner in which investors regularly use public equity markets to evaluate publicly traded stocks. While private equity markets are much less liquid than public equity markets and private equity investors generally hold their positions until liquidation without transacting in secondary markets, the pricing information inherent in secondary markets should still be useful for such investors. For example, bond markets are much less liquid than stock markets and bond investors often intend to hold their positions until the bond matures without transacting in secondary markets, yet bond investors still look to secondary markets to gauge the value of their portfolios through time. In addition, bond indices based on such secondary markets exist. Our intent is to develop similar

[1] Funds do report "Net Asset Values (NAVs)" to their investors, which are accounting-based valuations of the fund. These NAVs are adjusted to reflect the fund's actual value, but at any point in time, the gap between the NAV and the value of an investor's stake in a fund can be substantial.

indices for private equity funds. Further, any prevailing liquidity discounts in these secondary markets will impact expected returns only if liquidity risk is priced. Investigating the existence of a liquidity risk premium in private equity or understanding the impact of liquidity discounts on other moments of the return process is arguably easier if we first observe the return process itself. While the number of transactions on any particular fund is small, in aggregate the market contains enough trades to construct an index of overall returns. In addition, the indices can be used to value individual funds. We construct such indices for both buyout and venture capital funds, and use these indices to address a number of questions about the private equity market.

Absent a secondary market, fund returns are measured only over extremely long horizons and there is no simple way to know how much a private equity portfolio is worth at any point in time. For example, following the Financial Crisis of 2008 a number of investors believed they were "overweighted" in private equity, since their private equity positions were maintained on the books at stale NAVs while the market value of their stock holdings plummeted. Our results suggest that this view was naïve and that the value of private equity investments declined during 2008 by at least as much as public equity investments.

The primary challenge in constructing an index from secondary market data is accounting for the fact that every fund does not trade in every period, and many funds in our sample do not trade at all. In our data set of transactions, there are 3,404 fund transactions for 2045 funds from 2006 through 2017, implying that the average fund in our data trades 1.7 times in our sample. Moreover, there are many other funds that never trade through the intermediary that provided our sample. We take two general approaches to construct our indices in light of this challenge.

First, we follow the approach of Blume and Stambaugh (1983) and show that, under the assumption that funds transact with uniform i.i.d. probability, we can construct an index that tracks the prices of funds, even if they do not transact in our sample. Second, we account for the possibility that funds transactions are not random, and that the decision to transact in the secondary market could be related to fund market values or other characteristics. To account for such possible sample selection, we create a hedonic index using the approach of Heckman (1979). We estimate the parameters of an econometric model using observed transaction prices and each period create an inferred price for every fund, including those that do not transact, using a broad universe of funds. We then use these inferred prices to construct indices. We are careful to account for measurement error when estimating performance parameters by applying the correct bias adjustments (e.g. Scholes and Williams (1977) and Blume and Stambaugh (1983)).

The indices we develop based on secondary market transactions allow us to evaluate the risk-adjusted, net of fee performance of broad private equity portfolios. While there are a number of papers that have estimated private equity performance, none rely on secondary market data, which is ideal for measuring the risk and returns of securities. For this reason, our results differ in some regards to what has been reported in the literature. For example, current evidence suggests both buyout and venture funds outperform on a risk-adjusted basis (Cochrane (2005), Korteweg and Sorensen (2010), Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2016)). Results using our indices confirm that buyout funds outperform public markets, but suggest that venture capital funds do not.[2] We also find that NAV-based indices, such as the Burgiss index, tend to significantly understate the volatility of private equity as well as its covariance with other asset classes. Finally, our indices also allow us to value individual funds at any given point in time and to estimate the extent to which general partners over- or understate net asset values relative to market value over the course of the business cycle.

The paper proceeds as follows. In Section 2 we describe our contribution relative to the work of others. In Section 3 we describe our empirical methodology. In Section 4 we describe our data and results. In Section 5 we discuss some implications and applications. In Section 6 we discuss some institutional considerations. Section 7 concludes.

2. What Can We Learn from Private Equity Indices?

2.1. Prior Work Measuring Private Equity Risk and Return

[2] An important caveat is that it is well known that during our post 2000 sample period venture capital funds performed badly, while in earlier periods did extremely well.

The absence of an observable time series of market values at regular intervals for private equity has limited the ability of researchers to evaluate investment performance and value LP stakes in funds using standard empirical tools motivated by asset pricing theory. Basic parameters such as factor betas and alphas, volatility, correlations and average returns, have had to be estimated using non-traditional methods.

Prior studies about the investment performance of private equity can be broadly classified into one of four groups, depending on the type of data used. First, many studies use fund-level data on cash flows paid to and received by limited partners. Second, other studies use cash flows between private equity funds and their portfolio firms. Third, some studies use venture financing rounds and exit events (IPO, acquisition, and failure) which provide intermittent estimates of market value. Finally, some studies use other proxies for market value, such as NAV or the prices of similar publically listed securities.

Papers that use fund-level cash flows have relied on the PME approach, which measures the performance of a fund relative to the public equity market at the same time.[3] Recent studies that use relatively high-quality fund-level cash flow data find the PME for buyout funds to be in the range of 1.19-1.23 and for venture funds to be in the range of 1.06-1.36, suggesting that both of the major types of private equity funds beat the S&P 500 even after the fees that LPs pay (see Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2016)). Other studies use fund-level cash flows to estimate CAPM betas by estimating cross-sectional regressions of fund $IRRs$ on the $IRRs$ of the S&P 500 measured over the life of each fund (see Ljungqvist and Richardson (2003), Kaplan and Schoar (2005), and Driessen, Lin, and Phalippou (2012)). These papers generally find betas for both private equity types to be in the range of 1.08 to 1.23. Exceptions are Kaplan and Schoar (2005) who find a buyout beta of 0.41, and Driessen Lin, and Phalippou (2012) who find a venture beta of 2.73.[4]

[3] The *PME* approach was developed by Kaplan and Schoar (2005). Korteweg and Nagel (2016) suggest that (in the absence of secondary market data), the *PME* approach is a desirable way to measure private equity performance. $PMEs$ are calculated by discounting all cash flows of the fund at a rate equal to the total return on the S&P 500 index, and then dividing the future value of cash inflows by the future value of cash outflows. A fund with a PME above 1.0 therefore has outperformed the passive index over the evaluation period.
[4] Other papers that investigate fund-level cash flows include Chen, Baierl and Kaplan (2002), Phalippou and Gottschalg (2009), and Phalippou (2012).

Papers relying on cash flows between private equity funds and their portfolio firms generally estimate cross-sectional regressions of excess $IRRs$ on the excess $IRRs$ of factor portfolios in the cross section.[5] Among other things, these papers find CAPM alphas for buyout funds to be in the range of 9.3% to 16.3% with betas in the range of 0.95 to 2.3 (see Frazoni, Nowark, and Philippou (2012) and Axelson, Sorensen, and Stromberg (2014)). It is important to note that, in contrast to the estimates presented below, these studies estimate risk and return *gross of fees*. As emphasized by Axelson, Sorensen, and Stromberg (2014), the fees themselves vary positively with market returns, so gross of fee betas (and returns) will tend to be larger than those estimated net of fees.

A number of papers use valuations in venture financing rounds to measure the risk-adjusted performance of venture funds. These estimates are also gross of fees, so estimates of both risk and return will tend to be somewhat inflated relative to that received by investors. Since portfolio firms that receive more rounds of financing tend to be the better performing investments, these papers have to adjust for the sample selection implicit in their reliance of financing rounds. Papers that estimate the parameters of sample selection models find CAPM alphas for venture firms in the range of 32% to 38% with betas of 1.9 to 2.7 (see Cochrane (2005) and Korteweg and Sorensen (2010)). Other papers use venture financing events to create hedonic and repeated sales indices that account for sample selection and find alphas in the range of 4% and betas in the range of 0.6 to 1.3 (see Peng (2001) and Hwang, Quigley, and Woodward (2005)). An important caveat to these papers is that in venture capital deals, not all securities are equal, and since securities issued in later rounds tend to have more rights than those in earlier rounds, post-money valuations tend to overstate firms actual valuations (see Gornall and Strebulaev (2018)).

Finally, papers that use other proxies for market value provide additional evidence on risk adjusted fund performance. For example, Jegadeesh, Kraussl, and Pollet (2015) investigate fund performance based on market discounts relative to NAV observed for publicly listed firms that hold private equity. With a few

[5] The use of IRR is necessary since deal-level cash flows sometimes include intermediate cash flows occurring because of interim recapitalizations or equity injections.

exceptions, listed private equity firms tend to be general partners rather than particular funds, so the estimates presented in Jegadeesh, Kraussl and Pollet (2015) can be thought of as estimates of the risk and returns of these general partners. Other authors investigate fund performance using the stated NAV of funds (see Gompers and Lerner (1997) and Ewens, Jones, and Rhodes-Kropf (2013).

Overall, evidence in the literature suggests that both buyout and venture funds tend to perform well as investments. The average PME for both funds is usually estimated to be greater than 1, which implies that these funds outperform public markets. This outperformance could reflect positive alpha, the greater risk of private equity funds relative to the market, or both. Estimates of fund betas, however, are somewhat mixed, with some studies finding betas in the range of 1.0 and other studies finding betas well above 2.0.

2.2. Advantages of Private Equity Indices to Measure Performance

The approaches taken by prior studies to evaluate investment performance have limitations. For this reason, our results on private equity performance differ in some regards to what has been reported in the literature.

The shortcomings of PME and IRR are well known. While PME does help us understand the performance of private equity relative to a given benchmark portfolio, its ability to shed light on risk-adjusted performance is limited. For example, PME does not guide the researcher in choosing the correct benchmark portfolio, cannot account for multiple factor exposures, and cannot be manipulated to estimate the alphas and betas of factor models.[6] Moreover, standard asset pricing theory is built on the concept of returns. The IRR itself, in general, is not a return, is not unique, and may not exist.

Using the cross section of returns to estimate standard performance parameters for private equity can also be problematic for two main reasons. First, it is impossible to estimate parameters such as the volatility of an entire asset class using cross sectional data. Studies that estimate volatility using the cross-sectional dispersion of returns are estimating the expected volatility of a single fund or deal. For example,

[6] Kaplan and Schoar (2005) estimate alpha as $PME - 1$. Phalippou and Gottschalg (2009) estimate alpha as the constant that would need to be added to the chosen benchmark discount rate to drive the PME to 1.0. These are appropriate methods only if beta relative to the chosen benchmark portfolio is 1.0.

Cochrane (2005) uses cross-sectional variation in returns from financing events to estimate the average annualized volatility of the return from investing in a venture startup to be 107%. An investor with a broad portfolio, however, would be affected by the volatility of a portfolio of startup firms (through their impact on the funds' returns) rather than the expected volatility of a single position. Portfolio volatility primarily depends on the covariance structure across positions in addition the volatility of individual investments. Consequently, the volatility of a venture index is likely to be a better representation of the risk exposure faced by investors than the volatility of individual portfolio firms. Along these same lines, the investor will naturally be interested in the correlation of the portfolio with other asset classes, which cannot be estimated using cross-sectional data.

Second, as noted by Axelson, Sorensen, and Stromberg (2014), the irregular intervals over which returns or $IRRs$ are measured can be problematic. The IRR of a fund or deal is only observable when the fund or deal is complete, and hence, the IRR interval will vary widely in the cross section. The irregular intervals at which venture financing events occur will also cause variation in measured return horizons. Using $IRRs$ or returns compounded over irregular intervals can result in surprisingly large biases when estimating CAPM parameters. Axelson, Sorensen, and Stromberg (2014) simulate deal-level cash flows and estimate the CAPM using cross-sectional variation in $IRRs$. In some reasonable specifications, the beta is underestimated relative to the true beta, on average, by 116%. In other specifications, beta is overestimated by 123%.

To deal with irregular sampling studies often assume returns are generated by a continuous-time process and use log returns.[7] This great flexibility comes at a cost in terms of strong parametric assumptions that can have a meaningful influence on results. For example, a straightforward application of Ito's Lemma provides the necessary adjustment needed to transform the intercept in a standard factor regression using log returns into a continuous-time alpha:

[7] This is the approach taken by Chen, Baierl and Kaplan (2002), Cochrane (2005), Korteweg and Sorensen (2010), Frazoni, Nowark and Phalippou (2012), and Axelson, Sorensen, and Stromberg (2014). Also, see the discussion in Campbell, Lo, and MacKinlay (1997), pp. 363-64.

$$\alpha = \delta + \frac{1}{2}\beta(\beta - 1)\sigma_m^2 + \frac{1}{2}\sigma_\epsilon^2 \tag{1}$$

where δ is the intercept in the standard regression of excess log asset returns on excess log market returns, β is the slope coefficient in this regression, σ_m^2 is the variance of log market returns, and σ_ϵ^2 is the variance of the residual in this regression. Cochrane (2005) estimates $\delta = -7.1\%$ and $\alpha = 32\%$. The large difference in these parameters is driven by a large idiosyncratic variance. Cochrane (2005), in fact, estimates $\sigma_\epsilon = 86\%$. The caveat noted by many authors is that while the adjustment enacts non-trivial changes on the intercept, it is derived based on the strong parametric assumptions of the continuous-time CAPM.

Another issue when working with log returns is that funds and deals at times go bankrupt, and the log of zero is undefined. To solve this problem some studies censor the log $IRRs$ of bankrupt firms (or deals) to some negative finite number and explicitly account for censoring in the econometric model (Axelson, Sorensen, and Stromberg (2014)). Other studies create portfolios of funds or deals with returns that are never zero (Driessen, Lin and Phalippou (2012) and Frazoni, Nowark and Phalippou (2012)), while some studies do not discuss the issue and apparently remove these observations from the sample. In our Preqin sample of fund-level cash flows, we in fact find that 1.7% of buyout funds and 3.7% of venture funds exhibit cash flows with negative NPV regardless of the discount rate. Removing such funds from the sample leads to biased parameter estimates.

Given the difficulties of measuring performance using the cross section of $IRRs$ or returns, other authors create indices using venture financing events as intermittent estimates of market value. Peng (2001) and Hwang, Quigley, and Woodward (2005) develop hedonic and repeat sales indices using venture financing events and methods that are, in fact, similar to those we use to create our indices. Such indices can also be problematic, however, for three reasons. First, financing events represent prices at which an investor can get in to venture deals, but not at which the investor can get out. Second, in venture capital, not all shares are created equally; newly created shares in financing events give more rights than old shares so that implied valuations can be misleading (see Gornall and Strebulaev 2018). Third, returns from venture financing events are gross of fees, making it difficult to understand the return earned by limited partners that invest directly in private equity funds.

Jegadeesh, Kraussl, and Pollet (2015) take another approach to investigate fund performance based on market discounts relative to NAV observed for publically listed securities (Listed funds of funds and listed private equity) that are similar to standard private equity positions, but include some important differences. Large buyout firms such as Blackrock and KKR hold a variety of investments other than private equity. Jegadeesh, Kraussl, and Pollet (2015) include in their study any fund of fund that holds at least 50% of their capital in unlisted funds. Funds of funds also tack on an extra layer of fees that make it especially difficult to understand the return earned by limited partners that invest directly in private equity funds.

The indices we develop based on secondary market transactions enable us to evaluate the risk-adjusted performance of broad private equity portfolios using standard empirical methods that avoid some of the pitfalls of methods used in prior studies. We use our indices to create time-series of arithmetic returns quoted at regular intervals. In contrast to a single fund or deal, the index returns have relatively little idiosyncratic risk implying that parameter estimates should be less sensitive to using either log or arithmetic returns. (We in fact find that our estimates of alpha are nearly identical whether we use simple returns, log returns, or the continuous time adjustment discussed above.) The indices we create include bankrupt funds. Moreover, we build the indices from data on the actual secondary market transactions of private equity positions, net of fees. Together, these advantages enable us to obtain more reliable estimates of risk-adjusted performance of private equity from the perspective of the limited partner, the investor. Finally, the indices also allow us to investigate variation in the market value of private equity over time.

3. Methods

Private equity returns are a function of transaction prices, fund contributions and fund distributions. We observe quarterly distributions and contributions for a large universe of funds. In contrast, we observe market prices for a smaller subset of funds since no fund transacts in every quarter and some funds never transact in our sample. In addition, transactions that do occur are highly non-synchronous. Our observed index returns therefore contain measurement error coming from two sources: non-trading and non-synchronous trading.[8] Non-trading, after accounting for any sample selection in the funds that trade, induces i.i.d. measurement error in our index returns similar to the setting investigated by Blume and Stambaugh (1983). Non-synchronous trading, induces spurious autocorrelation and cross-autocorrelation with the market (see for example, Scholes and Williams (1977), Lo and MacKinlay (1990)). In addition, both sources of measurement error induce biases in estimated variances and covariances using observed index returns. We now show how we construct our indices and correct for biases in estimated moments that arise from measurement error.

3.1 Index Construction

Let $P_{i,t}$ denote the price of a \$10 million commitment to private equity fund i as of the end of quarter t and suppose at the end of quarter $t-1$ we acquire a \$10 million commitment to each of N different equity funds. If we hold for one period and then sell all of our positions at the end of quarter t, the log quarterly buy-and-hold return for the portfolio is:

$$r_t = \log\left(\sum_i P_{i,t} + D_{i,t} - C_{i,t}\right) - \log\left(\sum_i P_{i,t-1}\right) \qquad (2)$$

where $D_{i,t}$, $C_{i,t}$ represent the total distributions and contributions associated with our exposure to fund i during quarter t. For convenience we can write equation (2) as:

$$r_t = \log(\bar{P}_t + \bar{D}_t - \bar{C}_t) - \log(\bar{P}_{t-1}), \qquad (3)$$

where an overline represents a simple average. Equation (3) defines the log return of a price-weighted portfolio or index of the private-equity positions. The analysis of this section can be extended to portfolios with general weights by defining $P_{i,t}$ to be the price of the appropriate sized position in fund i. As discussed above, the moments and estimated performance parameters of our index are very similar for log returns and

[8] Non-trading in a given period can be modeled as extensive non-synchronous trading as in Lo and MacKinlay (1990) if all securities trade at some point in time. In our data, however, some funds never transact.

for arithmetic returns, in part because idiosyncratic variation in the index is small. We choose to use log returns to explain our methodology for estimating $\bar{P}_t$ and the moments of r_t since doing so facilitates the exposition.

Assume for the time being that we observe a transaction price for all N funds in quarter t, but that funds do not transact at the same time. Instead, following Scholes and Williams (1979), suppose that we assign each fund a random transaction time, $t - s_i(t)$, with $0 < s_i(t) < 1$ and $s_i(t)$ distributed i.i.d. across time for fund i, and potentially correlated across funds. We denote the observed transaction price for fund i at the end of quarter t as $P_{i,t-s(t)}$ and the end-of-quarter price as $P_{i,t}$.

Now assume that in quarter *t* a set of $k_t < N$ funds transact, and for now assume that funds transact with independent uniform probability at their appointed transaction times. The observed transaction price for any fund can always be written as the product of the average across all *N* funds, $\bar{P}_{t-s(t)}$, and a fund-specific scaling constant, $\left(1+\delta_{i,t}\right)$:

$$P_{i,t-s(t)} = \bar{P}_{t-s(t)}(1+\delta_{i,t}), \tag{4}$$

where $\delta_{i,t} > -1$ and the population average of $\delta_{i,t}$ across all *N* funds is identically equal to zero, $\bar{\delta}_t = 0$. Let $\bar{\delta}_{k,t}$ denote the average value of $\delta_{i,t}$ across the k_t funds that transact. The observed estimate of the average price using these k_t funds, $\bar{P}^k_{t-s(t)}$, is given by:

$$\bar{P}^k_{t-s(t)} = \bar{P}_{t-s(t)}(1+\bar{\delta}_{k,t}), \tag{5}$$

where $\bar{\delta}_{k,t}$ is the average value of $\delta_{i,t}$ across the k_t funds that transact. If funds transact with independent uniform probability, then $\bar{\delta}_{k,t}$ is independent across time and mean zero.

Using equation (5) we can write the observed average log price across all funds as:

$$\log(\bar{P}^k_{t-s(t)}) = \log(\bar{P}_t) + \log\left(1+\bar{\delta}_{k,t}\right) - \tilde{r}_t, \tag{6}$$

where

$$\tilde{r}_t = \log(\bar{P}_t) - \log\left(\bar{P}^k_{t-s(t)}\right) \tag{7}$$

and $\bar{P}_{i,t}$ represents the average price using (unobserved) end-of-quarter *synchronous* prices across *all* funds. The random variable $\tilde{r}_t$ is similar to the portfolio return from buying each of the k_t funds at their assigned transaction time, and selling them all simultaneously at their end-of quarter market values. Before deriving the implications of (6) for the moments of observed portfolio returns, we first show that a similar result holds even when funds do not transact with uniform i.i.d. probability.

Because some types of funds are more likely to transact than others, our estimate of $\bar{P}_t$ may contain sample selection bias if we simply take a simple average of observed prices. We therefore develop a hedonic model. Papers that develop hedonic indices generally do so to estimate the price change for a single good or basket of goods with constant characteristics over time using observed prices for differentiated goods over time (see, for example, Gatzlaff and Haurin (1998), Pakes (2003) and Hwang, Quigley, and Woodward (2005)). Our objective is to understand the price changes of a *portfolio* of differentiated goods over time when some transaction prices are not observed. We therefore take a slightly different approach than these authors.

Suppose that $k_t < N$ funds transact in period t and let $\boldsymbol{\pi}_{k,t-s(t)}$ denote a vector of observed scaled market prices with element i defined as $\pi_{i,t-s(t)} = P_{i,t-s(t)}/NAV_t$. Let $\boldsymbol{X}_{k,t}$ denote a $k_t \times p$ matrix of p characteristics observable at the end of period t for each fund that transacts, and let $\boldsymbol{\theta}_{k,t}$ denote the $p \times 1$ vector of parameters estimated using this data by running the following regression,

$$\boldsymbol{\pi}_{k,t-s(t)} = \boldsymbol{X}_{k,t}\boldsymbol{\theta}_{k,t} + \boldsymbol{z}_{k,t}\ , \tag{8}$$

where $\boldsymbol{z}_{k,t}$ denotes a zero-mean vector of error terms.

Now assume that while we do not observe $\pi_{i,t-s(t)}$ for every fund in a given universe, we do observe the explanatory variables in the regression specified in (8) for every fund in that universe in the $N \times p$ matrix $\boldsymbol{X}_t$. We can use the estimated coefficients from the regression above to obtain an estimate of $\pi_{i,t-s(t)}$ for every fund at the end of quarter t, stacked in the $N \times 1$ vector $\boldsymbol{\pi}_{f,t-s(t)}$:

$$\boldsymbol{\pi}_{f,t-s(t)} = \boldsymbol{X}_t\boldsymbol{\theta}_{k,t}. \tag{9}$$

We can then estimate an $N \times 1$ vector of "fitted prices" for all N funds in the chosen universe at the end of quarter t:

$$\boldsymbol{P}_{f,t-s(t)} = \boldsymbol{\pi}'_{f,t-s(t)}\boldsymbol{NAV_t}, \tag{10}$$

where $\boldsymbol{NAV_t}$ is an $N \times 1$ vector of net asset values. Note that for the unobserved case in which $k_t = N$ and we have $\pi_{i,t-s(t)}$ for every fund, the average fitted price in (10) is *identical* to the average price of the *entire* fund population, $\bar{P}_{f,t-s(t)} = \bar{P}_{t-s(t)}$, because regression error terms are mean zero.[9]

Given that we observe prices for only $k_t < N$ funds, our estimate of the regression parameters will in general not be identical to the estimate using prices for all funds. Let $\boldsymbol{\theta}_t$ denote the vector of estimated regression parameters if all prices were observable, and let $\boldsymbol{\eta}_t = \boldsymbol{\theta}_{k,t} - \boldsymbol{\theta}_t$. Heckman (1979) develops methods (further discussed below) to help ensure that $E[\boldsymbol{\eta}_t] = \mathbf{0}$ independent of $\boldsymbol{X}_t$, $\pi_{i,t-s(t)}$, and $NAV_{i,t}$. It follows that the average fitted price using the k_t funds that transact is given by

$$\bar{P}_{f,t-s(t)} = \bar{P}_{t-s(t)}\left(1 + \frac{\bar{\boldsymbol{x}}'_{k,t}\boldsymbol{\eta_t}}{\bar{P}_{t-s(t)}}\right) \tag{11}$$

where $\bar{\boldsymbol{x}}_{k,t}$ denotes the vector of average explanatory variables across the k_t funds that transact, and the ratio $\bar{\boldsymbol{x}}'_t\boldsymbol{\eta_t} / \bar{P}_{t-s(t)}$ is mean zero and independent across time. If we let $\bar{\delta}_{k,t} = \bar{\boldsymbol{x}}'_t\boldsymbol{\eta_t} / \bar{P}_{t-s(t)}$ then the observed log average price can again be written as

$$\log(\bar{P}_{f,t}) = \log(\bar{P}_t) + \log(1 + \bar{\delta}_{k,t}) - \tilde{r}_t, \tag{12}$$

as in equation (6), with $\bar{\delta}_{k,t}$ independent across time and mean zero. The only difference between (6) and (12) is the source of measurement error in $\bar{\delta}_{k,t}$ that arises from the assumed process by which funds transact. In (6) we assume a subset of funds transacts at random leading to i.i.d measurement error in a simple average estimate of $\bar{P}_{i,t}$. In (12) we assume certain kinds of funds are more likely to transact. Using a regression to infer the prices of *all* funds which are then used to estimate $\bar{P}_{i,t}$, i.i.d measurement arises because our estimated regression parameters contain i.i.d. measurement error relative to the estimate using the entire universe of funds.

[9] To see this note that $\pi_{f,i} = \pi_i - z_i$, multiply both sides by $NAV_{i,t}$ and take an average across i. The average fitted price is equal to the average price only if $NAV_{i,t}$ is independent of the regression residual. We can in fact ensure this is true by including NAV as one of the explanatory variables in the regression.

Equations (6) and (12) imply we can write the observed log portfolio return, r_t^O, as

$$r_t^O = r_t + \Delta\bar{\delta}_{k,t} - \Delta\tilde{r}_{i,t} \tag{13}$$

where we use the first-order approximation

$$\Delta\bar{\delta}_{k,t} \approx \log\left(1 + \bar{\delta}_{k,t}\right) - \log\left(1 + \bar{\delta}_{k,t-1}\right). \tag{14}$$

If true fund returns, measured from the end of one quarter to the next, are i.i.d mean zero and $s_i(t)$ is distributed i.i.d. across time for each fund, then $\tilde{r}_{i,t}$ is i.i.d across time and correlated with r_t.

Using equation (13) we can derive the moments of the observed log returns:

$$\begin{aligned}
E[r_t^O] &= E[r_t] \\
Var[r_t^O] &= Var[r_t] + 2Var\left[\,\bar{\delta}_{k,t}\right] + 2Var\left[\tilde{r}_{i,t}\right] - 2Cov(r_t, \tilde{r}_t) \\
Cov\left[r_t^O, r_{m,t}\right] &= Cov\left[r_t, r_{m,t}\right] - Cov\left[\tilde{r}_t, r_{m,t}\right] \\
Cov\left[r_t^O, r_{t-1}^O\right] &= -Var\left[\bar{\delta}_{k,t}\right] - Var\left[\bar{\delta}_{k,t}\right] - Cov\left(r_t, \tilde{r}_{i,t}\right) \\
Cov\left[r_t^O, r_{m,t-1}\right] &= Cov\left[\tilde{r}_t, r_{m,t}\right].
\end{aligned} \tag{15}$$

where $r_{m,t}$ is the observed market return (e.g., the return on the S&P 500) with no measurement error. The first line of (15) indicates that measured portfolio returns are unbiased. The second line of (15) indicates that the observed portfolio variance may be either over- or understated (see discussion in Scholes and Williams(1979) and Lo and MacKinley (1990)). The third line of (15) indicates the covariance of the observed portfolio return with the market is understated due to the reduced contemporaneous overlap in these measured returns from non-synchronous trading. The fourth and fifth lines of (15) provide results for the observed autocovariance and cross-autocovariance with the market. These bottom two relationships can be used to correct the biases in the estimated variance and contemporaneous covariance. The bottom line of (15) uses the equality $Cov\left[\tilde{r}_t, r_{m,t}\right] = Cov\left[\tilde{r}_{t-1}, r_{m,t-1}\right]$ which follows from the assumption that both returns and $s_i(t)$ are distributed i.i.d. across time.

The moments defined in equation (15) enable us to derive unbiased estimates of the following moments for the portfolio return of all funds with synchronous trading,

$$E[r_t] = E[r_t^O] \tag{16}$$

$$Var[r_t] = Var[r_t^O] + 2Cov[r_t^O, r_{t-1}^O]$$
$$Cov[r_t, r_{m,t}] = Cov[r_t^O, r_{m,t}] + Cov[r_t^O, r_{m,t-1}].$$

The moments defined in (16) can be easily used to define unbiased moments of other parameters of interest, such as the CAPM alpha and beta, as well as the contemporaneous correlation between the fund portfolio return and the market.

3.2. Heckman Sample Selection Model

The hedonic index relies on the assumption that $E[\boldsymbol{\eta}_t] = \mathbf{0}$. Various factors determine which funds are be selected for transaction. If these factors are independent of transaction prices, then the OLS estimate of $\boldsymbol{\theta}$ using only the k_t funds that transact is unbiased, implying $E[\boldsymbol{\eta}_t] = \mathbf{0}$. On the other hand, if omitted variables are correlated with both fund selection and price, then the OLS estimate of $\boldsymbol{\theta}$ is biased. To see this assume we can model the transaction process as

$$\pi_{i,t} = \boldsymbol{x}'_{i,t}\boldsymbol{\theta} + \epsilon_{i,t} \tag{17}$$
$$y^*_{i,t} = \boldsymbol{z}'_{i,t}\boldsymbol{\gamma} + v_{i,t}$$
$$y_{i,t} = \begin{cases} 1 \text{ if } y_{i,t} \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

where $\boldsymbol{x}_{i,t}$ and $\boldsymbol{z}_{i,t}$ represent a set of characteristics for fund i in quarter t observable across all funds in the portfolio. The variable $y^*_{i,t}$ is a latent variable that describes when a transaction occurs, such that fund i transacts in quarter t if and only if $y^*_{i,t} \geq 0$. Since we observe which funds transact, we do observe $y_{i,t}$. We refer to the first equation of (17) as the *pricing equation*, and the second equation as the *selection equation*. The error terms $\epsilon_{i,t}$ and $v_{i,t}$ may be correlated, reflecting the possibility that unobservable characteristics are associated with both price and fund selection. For our purposes, since we are only interested in estimating the average price and not in any causal relationships, neither the pricing equation nor the sample selection equation need be causally identified.

The OLS estimate of $\boldsymbol{\theta}$ is biased if $\epsilon_{i,t}$ and $v_{i,t}$ are correlated since we observe the dependent variable, $\pi_{i,t}$ only for funds that transact, $y_{i,t}^{*} \geq 0$, and

$$E\left[\epsilon_{i,t} \middle| y_{i,t}^{*} \geq 0\right] = E\left[\epsilon_{i,t} \middle| v_{i,t} \geq -\boldsymbol{z}_{i,t}'\boldsymbol{\gamma}\right]. \tag{18}$$

The expected value of the OLS estimate of $\boldsymbol{\theta}$, using observed data, $E[\widehat{\boldsymbol{\theta}}]$, equals

$$E\left[\widehat{\boldsymbol{\theta}}\right] = \boldsymbol{\theta} + E[(\boldsymbol{X}'\boldsymbol{X})^{-\mathbf{1}}\boldsymbol{X}'\ ]E\left[\epsilon_{i,t} \middle| v_{i,t} \geq -\boldsymbol{z}_{i,t}'\boldsymbol{\gamma}\right]. \tag{19}$$

where $\boldsymbol{X}$ is the matrix obtained by stacking the row vectors $\boldsymbol{x}_{i,t}'$ for all i and for all t.[10] Unless $\epsilon_{i,t}$ and $v_{i,t}$ are independent, the OLS estimate of $\boldsymbol{\theta}$ is biased.

Heckman (1979) proposes a simple two-step approach to estimate the parameters of the model given in (17). We estimate these parameters by MLE. Monte Carlo experiments indicate MLE is often more efficient than the two-step approach.[11] In addition, MLE allows for straight-forward computation of robust asymptotic standard errors, is convenient for conducting standard model diagnostics, and imposes the natural restriction that $|\rho| \leq 1$ where ρ represents the correlation between $\epsilon_{i,t}$ and $v_{i,t}$. Regardless, we find that our results are virtually unchanged using either approach to estimate the parameters of the model given in (17).

If $\epsilon_{i,t}$ and $v_{i,t}$ are distributed normal,

$$\binom{\epsilon_{i,t}}{v_{i,t}} \sim N\left[\binom{0}{0}, \begin{matrix} \sigma_{\epsilon} & \rho\sigma_{\epsilon} \\ \rho\sigma_{\epsilon} & 1 \end{matrix}\right], \tag{20}$$

[10] In our empirical application we use the entire panel of fund prices and characteristics to estimate θ, rather than estimating the regression quarter-by-quarter as presumed in section 2.1. We do this because some quarters have relatively few transactions.

[11] See Puhani (2000) for a survey.

then it is well known that the log-likelihood function of the model given in (17) is

$$\mathcal{L}(\theta,\gamma,\rho,\sigma_\epsilon;\boldsymbol{x},\boldsymbol{z},\boldsymbol{\pi}) = \sum_{N_0}\log\left[1-\Phi\left(\boldsymbol{z}_{i,t}'\boldsymbol{\gamma}\right)\right] + \tag{21}$$

$$\sum_{N_1}\left[-\log\sigma_\epsilon + \log\phi\left(\frac{\pi_{i,t}-\boldsymbol{x}_{i,t}'\boldsymbol{\theta}}{\sigma_\epsilon}\right) + \log\Phi\left(z_{i,t}'\gamma + \frac{\frac{\rho}{\sigma_\epsilon}\left(\pi_{i,t}-\boldsymbol{x}_{i,t}'\boldsymbol{\theta}\right)}{\sqrt{1-\rho^2}}\right)\right]$$

where N_0 represents the set of observations over i and t for which no transaction prices are observed, and N_1 is the set of observations for which we do observe transaction prices.[12] We estimate the standard errors of the parameters using the quasi-maximum-likelihood approach of White (1982), which accounts for heteroscedasticity and any cross-sectional or time-series dependence, and can be valid even if the true density of $\epsilon_{i,t}$ and $v_{i,t}$ is not normal.

Semi-parametric identification of selection models requires a variable that is in the selection equation but is independent of the error term in the pricing equation (see Heckman (1990), Leung and Yu (1996), Andrews and Schafgans (1998), Korteweg and Sorensen (2011), and Wooldridge (2010) pp. 803-808.) Although parameter estimates are still consistent with no exclusion restriction under the parametric assumptions of the model (specifically, the assumption of normality), they are less efficient and more sensitive to model assumptions. Keep in mind the objective is to measure explained variation in fund prices, not to identify any casual effects.

We therefore incorporate an exclusion restriction in our model that is arguably independent of $\epsilon_{i,t}$ on theoretical grounds. Specifically, our exclusion restriction is the fraction of limited partners for a given fund that are pension funds. Pension funds are typically buy-and-hold investors with the main investment objective of matching the duration of liabilities. As such, we expect greater pension fund holdings to be associated with fewer fund transactions, or a lower propensity for fund selection. On the other hand, the characteristics of the limited partners are unlikely to be correlated with transaction prices.

[12] See, for example, Hall (2002).

4. Data and Results

4.1 Data

A large intermediary in the private equity secondary market provided us with their complete database on all the secondary market transactions intermediated by their firm.[13] The database identifies the fund name, the vintage, the total capital committed by the seller, the amount unfunded by the seller, the purchase price, and the transaction date for funds that transacted from June of 2000 through December of 2017. Since the database contains only five transactions before 2006, we eliminate these and conduct our analysis using transactions that take place between 2006 and 2017.

We first identify all buyout and venture funds in the transactions data using the "fund type" field and eliminate all other transactions. We then carefully clean the data as detailed in the paper appendix and pull the most recent transaction for each fund each calendar quarter. After cleaning the data, we are left with 3404 fund transactions for 2045 funds of which 1170 are buyout funds and 875 are venture funds. We refer to these data as the *transaction sample*.

We obtain data on other fund characteristics, such as calls, distributions, NAV, fund LP type, and size for a large universe of funds from Preqin. We narrow these data to buyout and venture funds using the "category type" field in Preqin, as detailed in the appendix. Within each calendar quarter we sum all contributions and distributions (separately) for a given fund. We eliminate any fund/quarters for which NAV is less than zero and also remove records for which fund size (total capital committed) or LP type is missing. We also eliminate any funds for which we do not have cash flow data since the fund's inception.

A reporting lag causes our data to be missing information for the last quarter of 2017, and hence, our data from Preqin extend from the first quarter of 2006 through the third quarter of 2017. After cleaning the data in this manner, we are left with quarterly information on 1879 unique funds, of which 979 are buyout and 897 are venture. We refer to these data as the *Preqin universe.*

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[13] See Nadauld, Sensoy, Vorkink, and Weisbach (2018) for a detailed description of this database for a somewhat shorter period than is used here.

We then carefully merge the transactions sample with the Preqin universe, some of which is done by hand. Details on the merging process are given in the appendix to this paper. In all, we identify 524 matching funds (294 buyout and 230 venture) in both databases which for which 1,246 transactions occurred from 2006 through 2017. We refer to these data as the *merged sample*. Finally, we also consider the subsample of funds in the merged sample that are four to nine years old, and call this the *fairway merged sample*.[14] Fairway transactions represent the most commonly traded group of transactions, so are a useful subsample for comparisons of deals.

Table I reports summary statistics for the sample. Panel A contains the statistics for buyout funds and Panel B for Venture funds. For this table we break apart the Preqin universe into the merged sample and its complement. The complement sample contains all quarter-fund observations in Preqin for which no transactions occurred. We also break apart records for funds that are four to nine years old in the Preqin universe into a *fairway merged sample* (the intersection of the set of funds in the Preqin universe that are four to nine years old with the transactions sample) and its complement.

The first three rows of each panel report the mean, first quartile (Q1) and third quartile (Q3) for transaction prices as a fraction of NAV, $\pi_{i,t}$. Funds on average transact at a discount, indicative of the low liquidity in these markets. Discounts are smaller for fairway transactions. The average $\pi_{i,t}$ for buyout funds is generally 0.82 to 0.83 but for fairway transactions is 0.89. Similarly, the average $\pi_{i,t}$ for venture funds is generally 0.80 to 0.83 but for fairway transactions is 0.92. Among venture fairway transactions, the third quartile for $\pi_{i,t}$ is 1.21, suggesting that many venture funds transact at a premium to NAV.

Funds that transact are generally larger than average. The average fund size in the buyout merged sample is about 4.5 billion, indicating these funds on average are larger “mid-market funds” as loosely defined by Axelson, Sorensen, and Stromberg (2014).[15] The average fund size in the buyout compliment

[14] This term comes from conversations with practitioners and refers to deals that are “in the fairway”, meaning that they are fairly typical transactions. Most readers can probably correctly infer which sport these practitioners like to play on weekends.
[15] Axelson, Sorensen, and Stromberg (2014) define large cap funds as funds with total committed capital exceeding USD 5 billion, and mid-market funds as funds with sizes between USD 500 million and 5 billion.

sample is about \$1.6 billion, indicating these funds on average are smaller "mid-market funds". Similar patterns are found for venture, though venture funds in our data are about 80% to 90% smaller than buyout funds.[16]

The average fund age of transacting funds tend to be around eight to nine years in our data, and average $PMEs$ tend to be in the range of 1.12 to 1.17 for buyout funds and 0.96 for venture funds. We calculate the PME for each fund using all cash flows up to the most recent date for which we have cash flow data in Preqin, using NAV as the terminal value for funds that have not liquidated.

Figure 1 reports the number of transactions per quarter for the merged sample. Table 1 indicates the average number of transactions in the merged sample per quarter is about 17 for buyout funds and about 11 per quarter for venture funds.

4.2. Constructing the Indices of Private Equity Performance

4.2.1 Naïve Indices

If funds transact with i.i.d uniform probability, they constitute a representative sample from the population of funds for the given quarter and an unbiased estimate of the price-weighted portfolio of all funds is give by

$$r_t^O = \frac{\bar{P}_{k,t-s(t)} + \bar{D}_t - \bar{C}_t}{\bar{P}_{k,t-1-s(t-1)}} - 1 \,. \tag{22}$$

Equation (22) is the arithmetic analog to equation (2). As mentioned above, we get very similar results using either log or arithmetic returns. We refer to indices created in this manner as *"naïve"* indices, since they ignore the potential for any sample selection. Naïve indices are price-weighted. The return on a naïve index is the same as that of a portfolio strategy that uses all capital at the end of each quarter to buy an equal

[16] We are missing *Size* for any funds in the transactions sample post 2014 that are not in the merged sample, caused by a data update that failed to include this field.

sized commitment to each fund and then liquidates at the end of the subsequent quarter after collecting distributions and paying out calls.

One way to express the average observed price, $\bar{P}_{k,t-s(t)}$, is:

$$\bar{P}_{k,t-s(t)} = \bar{\pi}_{k,t-s(t)}\overline{NAV}_t + Cov(\pi_{k,t-s(t)}, NAV_t). \tag{23}$$

An advantage of (23) is that it enables us to use the information in all $NAVs$ regardless of whether funds transact or not to estimate average price. While NAV is not the market price it is likely to contain some pricing information. We therefore use all funds in the transaction sample that are 4 to 9 years old to estimate both $\bar{\pi}_t$ and $Cov(\pi_t, NAV_t)$ and the Preqin universe to estimate $\overline{NAV}_t$.

We estimate both $\bar{\pi}_t$, $\overline{NAV}_t$, and $Cov(\pi_t, NAV_t)$ by quarter. In our sample of transactions for funds 4 to 9 years old there are two quarters for which we observe only a single buyout transaction. When computing the buyout naïve index, we use the covariance estimated from the previous period for these two quarters. In this sample there are also eight quarters for which there are zero venture transactions, making it impossible to estimate the naïve venture index for these quarters. To create the venture index we therefore first create a price-weighted index using all funds. For this total naïve index we compute the fraction of capital invested in buyout funds each quarter, and take an average across quarters, $\bar{w}_b$. We then compute the venture index return each quarter as:

$$r_{nv,t} = \frac{r_{tn,t} - \bar{w}_b r_{nb,t}}{1 - \bar{w}_b} \tag{24}$$

where $r_{tn,t}$ represents the return on the total naïve index and $r_{nb,t}$ is the return on the buyout naïve index.

The naïve index is naturally price-weighted. Other weighting schemes that allocate capital according to size or allocate capital equally across all funds are impossible to compute using the naïve approach, as price is not all observable across all funds at the beginning of each period to compute weights.

4.2.2 Hedonic Indices

To implement the sample selection model given in (17) we need to take a stand on the explanatory variables $\boldsymbol{x}_{i,t}$ and $\boldsymbol{z}_{i,t}$. Table 2 lists the explanatory variables we consider. The first 6 rows of Table 2 list the state variables we consider, or variables that are the same across funds and only vary across time. The last 6 rows of Table 2 list the fund-specific variables we consider that vary across funds and some of which vary across time. We compute hedonic indices for both buyout and venture using the Heckman (1979) sample selection model as described in section 3.2. For comparison we also compute the hedonic indices using simple OLS. We compute size-weighted, price weighted, and equally-weighted versions of the hedonic indices.

4.2.3 Other Indices

For comparison we also compute indices using NAV as an estimate of market value. For example, we estimate arithmetic returns for NAV-based indices as

$$r_{N,t} = \frac{\overline{NAV}_t + \bar{D}_t - \bar{C}_t}{\overline{NAV}_{t-1}} - 1 \tag{25}$$

where $\overline{NAV}_t$ is the average NAV across funds at the end of quarter t, and $\bar{D}_t, \bar{C}_t$ represent average distributions and calls from the end of quarter $t-1$ to the end of quarter t. We also examine the return properties of the buyout index created by Burgiss.

4.3 Results

Table 3 presents our estimates of the parameters for the sample selection model using both buyout and venture funds. We estimate these models using the merged sample as highlighted in Table 1. Panel A reports estimates of the pricing equation, and Panel B reports estimates of the selection equation. We estimate the pricing equation using both OLS and the Heckman sample selection model.[17]

[17] We estimate the model using the entire panel of data for either buyout or venture funds, rather than period by period as presumed in section 3.1, since some quarters contain relatively few transactions.

Panel B indicates a number of variables are associated with fund selection. Looking at results for the state variables, funds are more likely to transact when public equity markets are valued highly, as measured by the aggregate market-to-book ratio for equities (MB_t), and when private equity the average fund PME ($MPME_t$) is high. The coefficient on MB_t for buyout funds is 0.26 with a t-statistic of 2.3, and the coefficient on $MPME_t$ is 2.15 with a t-statistic of 2.9. Both coefficients are even higher for venture funds. Both funds types are also more likely to transact when the valuation confidence index is high, and when the crash confidence index is low.

Results for fund-specific variables suggest that fund size is very important for fund selection. Larger funds are much more likely to transact. The coefficient on $LSIZE_i$ for buyout funds is 0.42 with a t-statistic of 26.4, and for venture funds is 0.40 with a t-statistic of 16.7. In addition, fund age is also important for fund selection. The coefficients on both $AGE1_{i,t}$ (dummy that equals 1.0 if fund is less than 4 years old) and $AGE2_{i,t}$ (dummy that equals 1.0 if fund is 4 to 9 years old) are negative and significant for both buyout and venture. Funds older than 9 years (the group excluded in the age-dummy-variable classification) are most likely to be chosen for transaction. In addition, private equity funds held by pension funds are less likely to transact for both buyout and venture. The coefficient on $PENSION_i$ (fraction of LPs that are pension funds) is $-0.38\ (0.17)$ for buyout (venture) with a t-statistic of $-4.8\ (-2.2)$. The negative significant relationship between $PENSION_i$ and the likelihood of transacting is consistent with our ex-ante prediction for the exclusion restriction. Given that $PENSION_i$ is unrelated to price, our parameter estimates are less sensitive to the parametric assumptions of the Heckman (1979) model.

The results in Table 3 Panel A for the pricing equation indicate that the scaled market price ($\pi = Price/NAV$), the dependent variable in our pricing model, tends to be higher for both buyout and venture funds when aggregate equity volatility (VOL_t) is high, when the valuation confidence index ($VALUE_t$) is low (the market looks *overvalued* from the viewpoint of institutional investors), and the crash confidence index ($CRASH_t$) is high (a severe market crash is *not* likely from the viewpoint of institutional investors).

Other results for state variables on the pricing equation are mixed for buyout and venture funds, indicating differences in these markets.

Of the fund-specific variables, π tends to be higher for funds with higher valuations as measured by $NAV_{i,t}$, for funds with higher $PME_{i,t}$ and for fairway transactions ($AGE2_{i,t} = 1$). We observe higher valuations for funds that are 4 to 9 years old, in part, because such funds are likely to have more capital invested than younger or older funds. The bottom row of Panel A of Table 3 indicates that our simple pricing equation has a 33% R-square for buyout funds and an 18% R-square for venture funds using the Heckman approach.

The bottom rows of Table 3 in Panel B test the null hypothesis that there are omitted variables correlated with both fund selection and fund premiums, $H_0\colon \rho = 0$. The Heckman (1979) sample selection model helps ensure that our parameter estimates are unbiased even if $\rho \neq 0$. If $\rho = 0$ then even OLS estimates of the pricing equation are unbiased. The low t-statistic on ρ for buyout funds (-0.3) strongly indicates that ρ is insignificant from zero. The last three rows of Panel B report Wald, likelihood ratio, and lagrange multiplier test statistics and p-values to test this same null hypothesis. The χ^2 statistics have high p-values for buyout funds, again indicating again that we cannot reject the null that ρ is zero in this model. In contrast, the high t-statistic on ρ for venture funds (-7.0) and the low p-values on the Wald, and likelihood ratio test statistics indicate that we can reject the null that $\rho = 0$ for venture funds. These findings are consistent with the OLS estimates of the pricing equation in Panel A. For buyout funds, the Heckman and OLS test statistics are virtually identical. In contrast, the estimated parameters are somewhat different for the Heckman and OLS models using venture funds.

Tables 4 through 6 all report moment estimates for various indices we create and share the same format. [18] Tables 4 and 5 are for buyout funds while Tables 6 and 7 are for venture funds. Tables 4 and 6

[18] In this version of the paper we currently ignore measurement error in index values when estimating standard errors for index moments and compute standard errors by GMM and the delta method using the approach of Newey and West (1987) to account for any time series dependence. Future versions of the paper will develop inference following the subsampling methodology of Politis and Romano (1992, 1994) which will enable us to account for measurement error in index values.

use all data from 2006 through 2017 while Tables 5 and 7 use all data *excluding* 2008 and 2009 to investigate the sensitivity of our results to the financial crisis. In all four of these tables, Panel A reports moment estimates for transactions-based indices. These include hedonic indices that are size-weighted, price-weighted, and equally-weighted, as well as the naïve index that is price-weighted. Panel B reports moment estimates for comparable Preqin NAV-based indices, and Panel C reports the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 estimated using the merged sample to the merged fairway samples as reported in Table 1 and as explained in sections 3.1 and 3.2. To avoid reliance on the parametric assumptions needed when using log returns (see equation (1)), we report results for simple returns. Results are very similar using either log or simple returns. We create the NAV-based indices as in equation (25). Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1. For the naïve index, β and α are also bias adjusted. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index (correlation with market and Sharpe ratio) because doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not subject to the types of measurement error discussed in section 3.1 that motivate our bias adjustment.

The performance parameters in Panel A of Table 4 indicate that buyout funds have performed very well over our sample from 2006 to 2017. All four of our transaction-based indices have significant average returns ranging from 19% to 37%. In comparison, the average market return over this period is 9.43% using data from Ken French’s website. Betas for the transaction-based indices range from 1.47 to 2.10. Alphas are also economically significant in Panel A except for the hedonic price-weighted index in which case the alpha is zero. Unfortunately, all of our estimates of alpha are statistically insignificant in Panel A of Table 4. In Table 5, however, we show that excluding the financial crisis enables us to estimate alpha with much greater precision. Annualized volatilities for the broad index portfolios range from 41% to 82%

and Sharpe ratios range from 0.23 to 0.71. Autocorrelations for all 4 indices are all insignificant from zero.[19]

It is also interesting to consider the idiosyncratic risk of buyout funds. We estimate annualized market volatility over this period using data from Ken French's website to be 16%. Hence, for the size-weighted portfolio idiosyncratic volatility is $0.52^2 - \left(1.77(0.16)\right)^2 = 0.19$. While we are not aware of a paper that estimates idiosyncratic risk for buyout funds[20], this estimate is much lower than the average idiosyncratic risk of venture deals (0.86) documented by Cochrane (2005). An important difference of our results relative to others is that we are measuring the volatility of fund portfolios, rather than that of the average fund or deal. Low idiosyncratic risk suggests the necessary adjustment to transform log-return regression intercepts into alphas as in equation (1) is relatively low.[21]

We also estimate the hedonic buyout indices using the OLS coefficients of Table 3 and find the moment estimates to be virtually identical (not reported). This is not surprising given that the estimated OLS parameters in Table 3 are virtually to those of the Heckman sample selection model.

Panels B and C of Table 4 show that the Prequin NAV indices are significantly smoother over time than their corresponding transaction indices. Betas of the NAV indices are a whopping 80% lower that the transaction indices. For example, the beta of the hedonic price-weighted index is 2.10 while the beta of the NAV-weighted NAV index is only 0.35. The t-statistic for the difference is 3.53 (given in Panel C). If we bias adjust the NAV index as we do the transaction index, the beta of the NAV index increases to only 0.54, remaining 74% below that of the transaction-based index. Volatilities are also significantly lower for the NAV indices, ranging from about 40% to about 90% lower. For example, the volatility of the hedonic

[19] Non-synchronous trading generally leads to positive auto correlation in *portfolios* (see Scholes and Williams (1979) and Lo and MacKinley (1990)) while i.i.d measurement error leads to negative autocorrelation (see Blume and Stambaugh (1983)). These two effects could be nearly cancelling each other out in our estimates of autocorrelation. We do find positive, significant *cross autocorrelation* with the market (currently not reported), which is influenced only by non-synchronous trading under the assumptions of section 3.1.

[20] Ewens, Jones, and Rhodes-Kropf (2013) test whether idiosyncratic risk is priced in buyout and venture funds using NAV-based returns. They do not care about the level of idiosyncratic risk in private equity (which is not reported) but rather the association between idiosyncratic risk and expected returns.

[21] Using the moments of either log or simple returns as inputs to equation (1), we find the estimated adjustment to be about 3.5%.

price-weighted index is 0.82 while the beta of the NAV-weighted NAV index is only 0.08 with a t-statistic for the difference equal to 4.31. Figure 2 illustrates the size-weighted hedonic indices for both buyout and venture relative to their corresponding NAV indices.

Sharpe ratios and correlations with the market tend to be significantly higher for NAV-based indices, driven mostly by lower volatility. Autocorrelations for NAV-based indices also tend to be larger than those of the transaction indices, though these autocorrelations are unlikely to be the result of the kinds of measurement error discussed in section 3.1.

Table 5 reports moment estimates for buyout indices excluding the financial crisis. For the hedonic indices, we use the same parameter estimates for the pricing equation as in Table 4, and then simply omit years 2008 and 2009 when estimating index parameters. The general patterns documented in Table 4 also hold in Table 5: betas and volatilities are significantly lower for NAV-based indices, while Sharpe ratios, correlations, and autocorrelations tend to be significantly higher. One important difference is that we now measure significant alphas for all indices in the range of 13% to 25% with t-statistics ranging from 1.98 to 2.24. Betas and volatility are also lower in Table 5 than in Table 4. Omitting the financial crisis causes betas of transaction-based indices to drop by about 60%. In Panel A of Table 5 betas are in the range of 0.49 to 0.87. Betas of NAV-based indices also drop by 25% to 40%, ranging from 0.04 to 0.23 in Panel B. Because market volatility is lower post crisis, changes in betas post crisis are driven by changes in estimated covariances.

Low betas for buyout funds may appear to be a puzzle, given their high degree of leverage. Our indices are net-of-fees. Fees can reduce beta if the expectation of fees as a fraction of distributions negatively covaries with the market. Axelson, Sorensen, and Stromberg (2014) provide evidence that the non-linear fee structure of buyout funds can have a surprisingly large effects on betas, reducing them by 50% or more.

Table 6 reports moment estimates for venture fund indices where we see that venture funds do not perform as well as buyout funds over our sample period, either on an absolute or risk-adjusted basis. Here average returns are in the range of 3% to 11% (about the same as that for the market, 9.43%) and insignificant from zero. Betas, on the other hand, are all in the range of 1.02 to 1.43, implying that alphas tend to be slightly negative (but insignificant from zero). In contrast, alphas using the Preqin NAV-based indices for venture funds (Panel B) tend to be significantly positive in the range of 5% to 13%. Average returns for the NAV indices are also significant, in the range of 8% to 15%, but not much different than those for the transaction indices in Panel A in terms of magnitude. The significant positive alphas for the NAV indices are an artifact of their low betas. Similar to the buyout indices, we see that the NAV-based indices have significantly lower betas and volatilities than the transaction indices. Similar to buyout indices NAV betas are again about 80% lower than those of the transaction-based indices, and volatilities are about 40% to 90% lower than those of transaction indices. NAV indices also have significantly higher Sharpe ratios, correlations, and autocorrelations. The implied idiosyncratic risk of a broad venture fund portfolio is 0.23, again, much lower than the average idiosyncratic risk of venture deals (0.86) documented by Cochrane (2005).

When we build the hedonic indices using the OLS parameters from Table 3 in the pricing equation the moment estimates (not reported) are somewhat different, though the empirical patterns discussed above still hold. In any event, for the case of venture funds, we should be suspicious of OLS results given the strong evidence indicating that the model contains omitted variables correlated with both price and fund selection, $\rho \neq 0$, as discussed above. The purpose of estimating the Heckman (1979) sample selection model is to obtain unbiased estimators in this setting.[22]

In Table 7 we provide moments estimates for the venture indices after omitting the years 2008 and 2009 as in Table 5. Again the results suggest that venture funds have not outperformed over our sample period. Alphas are all insignificant and close to zero, with the exception of the alpha for the hedonic equally-weighted index, where the point estimate is 11% with a t-statistic of 1.71. This alpha is somewhat suspect, however, as it is based off an estimate of beta that is negative (-0.16). Betas and volatilities are again lower post crisis for both NAV and transactions indices. NAV betas are about 40% lower while

[22] We are happy to share OLS results upon request. We may include these in the appendix in a future draft.

transactions betas are about 40% to 70% lower. Looking at Panel C we see again that betas and volatilities estimated using transactions-based indices tend to be significantly higher than corresponding estimates using NAV-based indices, while Sharpe ratios, correlations, and autocorrelations tend to be significantly higher for NAV-based indices.

In Tables 4 through 7, we find that results for the Naïve index are at least in the same range as parameter estimates for the hedonic index. For example, in the case of buyout funds using the full sample, the beta is 1.47 and the alpha is 15%, while the corresponding estimates using the hedonic index are 1.77 and 21%. Since naïve index results are model free, these results are supportive evidence that our results using the hedonic index are not driven by a peculiar feature of our model. Volatility for the naïve index may be understated, however, since we currently do not bias correct this parameter.

In Table 8 we compare the size-weighted hedonic index and the corresponding Preqin NAV-based index to the Burgiss index over the period from 2006-2017. The Burgiss index is a NAV-based index for buyout funds often used by institutions to analyze the performance of private markets. Results for the hedonic and NAV indices in Table 6 are identical to those reported in Table 4. Relative to these indices, the Burgiss index exhibits significantly lower average returns and alphas. For example, the average return for the NAV index is 26% while the alpha for the Burgiss index is 10%. The difference is statistically significant with a t-statistic of 1.97. The beta of the Burgiss index, however, is quite similar to the beta of the NAV index (0.45 versus 0.30), and the volatility of the Burgiss index is considerably lower than that of the NAV index (0.09 versus 0.30). This low volatility results in significantly higher correlation with the market for the Burgiss index. Similar to the Prequin NAV index , the Burgiss index is much smother over time with lower beta and volatility than the transaction-based index.

5. Implications and Applications

5.1 Economic implications.

Performance parameters estimated from the time series of market values can be helpful in making optimal portfolio allocation decisions, as they enable estimation of necessary parameters using standard empirical tools. Investors often turn to NAV-based indexes to estimate parameters such as volatilites and covariances. However, our estimates suggest that NAV-based indexes understate the covariances and volatilities of buyout and venture investments with the market. As previously reported, Tables 4 and 6 document market betas of 1.77 and 1.23 for buyout and venture funds, respectively, while estimates of market beta using NAV-based indexes are 70-80% smaller. These results are not surprising given the accounting practices used to calculate NAVs.[23]

The possibility that NAV-based indexes underestimate the true market covariance of venture and buyout funds could account for the increased allocation to these asset classes among institutional investors. As a means of calibrating how a NAV-based index could impact allocation decisions, we estimate the optimal long/short weights for each asset class, including the NAV-based time series index of buyout returns.[24] We then re-estimate optimal long/short weights with the base set of assets and our hedonic buyout and venture indexes. The sample period used to generate returns, variances, and covariances runs from 2006-2017, the years our indexes can be estimated.

Table 9 reports the proposed weights for each asset class, expected return, and standard deviations. Under the base asset case, the expected portfolio Sharpe ratio is exactly 1.00, driven largely by returns in equities and corporate bonds over our unique sample period. When the NAV-based buyout index is included, the estimated Sharpe ratio jumps to 1.59, and the optimization recommends a 72% weight in the NAV-based index, largely at the expense of investment in mid- and large-cap equities. When the NAV-based index is replaced with the size-weighted hedonic indexes, the portfolio Sharpe ratio drops to 1.25, with a recommended weight of 11% in the buyout index and 7% in the venture index. While we quickly caveat these results with concerns about the unique nature of our sample period, the results are consistent

[23] NAV-based indexes suffer from other limitations. Ljungqvist and Richardson (2003) note that inflating the value of NAV prior to the realization of cash distributions distorts the time profile of returns, which would also have the effect of distorting estimates of covariance with other asset classes.
[24] Our base set of assets include corporate bonds, commodities, real estate, 10-year treasuries, and small-, mid-, and large-cap stocks.

with the general intuition that NAV-based indexes likely understate the covariance of PE as an asset class with other investible asset classes.

Aside from portfolio allocation decisions, transaction-based indexes could be beneficial to help determine investment payouts. The beneficiaries of institutional investments, universities and pensioners in particular, rely critically on the cash flows generated by investments. Yet many endowments place restrictions on the availability of funds as a function of the value of the portfolio. In circumstances where endowments are underwater, for example, beneficiaries may be unable to access any money from the endowment. Given that an increasingly larger fraction of institutional money is being allocated to PE, a timely and higher-frequency understanding of PE's contribution to portfolio value could impact the availability of the cash flows to beneficiaries.

Finally, given that LPs are contractually obligated to meet GPs capital calls, better insight into the likelihood and timing of capital calls should be of interest to PE investors. The timing of capital calls has implications for cash management and fund performance in general. Robinson and Sensoy (2016) demonstrate that market conditions are correlated with the timing of fund cash flows, suggesting that marking the value of a portfolio to market using actual market prices could provide LPs with better insight into the timing and probability of future cash inflows and outflows.

5.2 Applications.

A price-based index can be used to assess changes in the market value of a private equity investment at a quarterly frequency. For fund i of vintage year j at time t, the fund's history of quarterly cash inflows and outflows could be combined with the quarterly returns of the hedonic indexes to calculate market values in the following way. Beginning with the first contribution in the amount C which occurs in quarter $q0$, represented as $C(q0)$, subtract any distributions D that occur in quarter $q0$, represented as $D(q0)$. At the end of the initial quarter $q0$, the book value, and absent extreme circumstances, the market value of the investment in the fund, V, should be $V(q0)=C(q0)-D(q0)$. Incorporating subsequent changes in market values via the index, the end of the next quarter market value could be calculated as $V1 = V0(1+r) + C(q1)$

–*D(q1),* where *r* is the hedonic-predicted return for a given fund (either buyout or venture). At the end of quarter 2, the market value could be calculated as *V2=V1(1+r)+C(q2)-D(q2).* This process could be repeated for each quarter to obtain a market value for any quarter between the origination date and the fund's liquidation date. In circumstances where the full history of calls and distributions is not available, the beginning value could be the first available NAV at time *t*. In this circumstance, value at time 1 could be calculated as *V1 = NAV(q0)(1+r) + C(q1) – D(q1).* The resultant market values at time *t,* represented as *V(T),* can be compared to NAVs at time t, denoted *NAV(T),* by taking the ratio of the two, *V(T)/NAV(T).* The resultant market-to-book ratio measures the differences between market and book values in each quarter.

We perform this calculation for each fund in our Preqin sample for every quarter between 2006 and 2017, the years our index can be estimated, and report results in Table 10. Because our hedonic indexes are estimated with transactions from funds that are between 4 and 9 years old, we only calculate market values for 4-9 year old funds. Following the procedure described above, beginning in Q1 of the fifth year for each fund, we use the fund-specific predicted quarterly return and fund-specific calls and distributions to calculate a market value for each fund. For the purposes of reporting quarterly average market-to-book ratios, we sum market values for each fund in a quarter and divide by the sum of fund NAVs in the quarter. Because each fund in our sample is a different age in calendar time, we report results by vintage year, beginning with 2006 vintage funds through 2017 funds.

Panel A of Table 10 reports the Q4 average market-to-book ratio for buyout funds. Averages for each of the vintages are reported across the bottom of the table. Average market-to-book ratios range from a low of 0.91 for the 2003 vintage of funds to high of 1.24 for 2008 vintage funds. Market-to-book ratios are considerably lower during the crisis, ranging between 0.65-0.76 in 2008, for the 3 vintages that were old enough for our hedonic estimation. Funds that invested during the financial crisis, 2007 and 2008 vintages, did so presumably at lower valuations, report higher average market-to-book ratios.

Panel B reports market-to-book ratios for venture funds that are lower than the market-to-book ratios reported for buyout funds. This result could be expected given the lower average returns estimated from the hedonic venture index as compared to the hedonic buyout index. The market-to-book ratios for venture funds were less impacted by the financial crisis; ratios reach 0.8, 0.8, and 0.92 for the three age-eligible vintages. The 2004 and 2005 vintages report the highest average market-to-book ratios, with 2009 vintage funds reporting the lowest.

Individual funds could mark their values to market using the hedonic approach in one of two ways. The most accurate approach would be to generate fund-specific market values using the hedonic-estimated coefficients applied to the fund's attributes. A more general approach could involve multiplying a fund's NAV by the average market-to-book ratio of the industry.

6. Institutional Considerations

6.1 Portfolio Transactions in the Secondary Market

A unique feature of the PE secondary market is the fact that individual funds are frequently sold as part of a larger portfolio transaction. For example, a seller might offer to sell their ownership stake in five unique funds, hoping to sell a portfolio of holdings in one large transaction. In a portfolio transaction, the buyer submits an offer price for the entire portfolio of funds, and the buyer and seller enter into a contract to eventually transfer ownership based on the portfolio offer price. Given that the construction of our price index relies on the market prices paid for individual funds, rather than one price for a portfolio of funds, it is important to consider the economics that govern how prices get assigned to individual funds in a portfolio transaction to determine whether and how portfolio transactions could influence the index.

Once a buyer and seller are in contract to transfer ownership of the portfolio of funds, the process moves to a second phase. During the diligence process, buyers assign prices to each of the individual funds in the portfolio, subject to the constraint that the size-weighted average of the individual prices equals the winning offer price. The price allocation process can be nuanced because although the buyer bids on the full portfolio, they may in reality only have strong demand for certain funds in the offered portfolio. The buyer's assignment of prices to individual funds will reflect their demand for those funds. High prices are assigned to funds the buyer most demands and lower prices are assigned to funds they demand less, again, subject to the constraint that size weighted-average prices equal the full portfolio bid. Conversations with industry experts indicate that there are times when prices allocated to individual funds result in certain of the funds being excluded from the final transaction. Thus, the prices assigned to individual funds are a reflection of demand for the funds, albeit filtered through the portfolio purchasing process.

While the assignment of prices to individual funds is ultimately a reflection of demand, the concern is whether prices determined through a portfolio process are somehow systematically different than single-fund sale prices. Any bias in our index stemming from portfolio transactions would have to display time series properties given that our object of interest from the index is quarter-over-quarter returns. The fraction of portfolio transactions each quarter is volatile, but shows no consistent trend quarter-over-quarter.

6.2 Management Fees

Management fees in buyout and venture funds can be large, most frequently 2%-of-committed capital in addition to 20% carried interest. The impact of fees on estimates of alphas and betas for buyout and venture investments is an open question. Most estimates of beta are calculated net-of-fees. One notable exception is Axelson, Sorensen, and Stromberg (2014). Using a sample of deal-level buyout cash flows, they estimate gross-of-fee and net-of-fee betas, concluding that fees reduce estimates of beta by about one-third, from 1.80 to 1.34. Our index should be considered neta-of-fees, since secondary market investors assume fee liability in a transaction. Because index returns are calculated quarter-over-quarter, the key consideration for the impact of fees on our estimates of beta depends on how expected cash flows, including fees, map into prices in consecutive quarters. For fees to bias our index-based estimates of beta, it would have to be the case that expectations of fees *as a fraction* of total distributions over the life of the fund increase as of $t+1$ compared to expectations as of t. Innovations in expected fees relative to distributions would cause a reduction in price at $t+1$ relative to price at t, lowering returns as measured from t to $t+1$. Carried interest is often contingent on meeting performance hurdles, and fund performance is likely to be correlated with the market. As a result, innovations in price that could be attributed uniquely to innovations in expected fees likely have the effect of dampening our index's correlation with the market.

Understanding bias on account of fees is further complicated by a host of institutional features of buyout and venture investments that can alter GP incentives and behavior, and thus the timing of distributions. Issues such as the recycling of management fees, GP catch-up clauses that alter cash flow waterfalls, and claw-backs, to name a few. Again, signing bias on account of these contractual features requires taking a stand on innovations from t to $t+1$ in expected fees relative to distributions. Some are easier to sign than others. Claw-backs, the contractual right of LPs to claw-back fees if GPs over-claimed performance fees early in funds life, could also have the effect of dampening betas. Innovations in expectations of receiving refunded fees should raise prices in $t+1$ relative to t, all else equal, and innovations in expected claw-backs are more likely in depressed markets. The effects of recycling of management fees and catch-up clauses are more ambiguous.[25]

Finally, we note that a companion paper using the same secondary market transaction data, Nadauld, Sensoy, Weisbach, and Vorkink (2017), discusses a number of other institutional details that could influence prices in the secondary market. Each of the considerations discussed in this paper and the companion paper are not expected to create bias in the estimates of the indexes.

7. Conclusion

Measuring the performance of private equity investments (buyout and venture) is typically only possible over long horizons because the return on a fund is only observable following the fund's final distribution. We propose a new approach to evaluating performance using actual prices paid for funds in secondary markets. We construct indices of buyout and venture capital returns using a proprietary database of secondary market prices between 2006 and 2017. We find strong evidence that buyout funds outperformed public equity markets on both an absolute and risk adjusted basis over this period. In contrast,

[25] GPs have incentives to recycle management fees back into investments because doing so increases the amount of invested capital at work. However, innovations in expectations about the timing and likelihood of GPs' recycling of fees is not obvious. A similar argument can be made for the cyclicality of innovations in the expected fee impact of GP catch-up provisions.

venture funds performed about as well as pubic equity markets with alphas that are insignificant from zero. We also find that our transaction-based indices exhibit significantly higher betas and volatilities, and significantly lower Sharpe ratios and correlations with public equity markets relative to NAV-based indices built from Preqin and obtained from Burgiss. There are a number of potential uses for these indices; in particular, they provide a way to track the returns of the buyout and venture capital sectors on a quarter-to-quarter basis and to value illiquid stakes in funds.

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Appendix A: Data Details

We first classify funds in the transactions data as either buyout or venture funds based on the specified fund type. Specifically, we classify funds as buyout funds that are labeled as "Buyout," "Small Buyout," "Mid-Cap Buyouts," "Mid-Cap Buyout," "Buyout/Growth," "Mega Buyouts," "Large-Cap Buyouts," and "All Private Equity". We classify funds as venture funds that are labeled "Venture," "Venture (General)," "Early Stage," "Early Stage VC," "Early Stage: Seed," "Early Stage: Start-up," "Expansion/Late Stage," and "Growth, and Mezzanine". After classifying funds as either buyout or venture and eliminating all other transactions we have 5214 fund transactions from 2006 through 2017 of which 3277 are for buyout funds, and 1937 are for venture funds.

We then clean the transactions data as follows:

1) Eliminate transactions with a price less than zero.
2) Eliminate transactions with a NAV less than zero.
3) Eliminate transactions that have the same price for every fund in the portfolio transaction.[26]
4) Eliminate two transactions that appear to be obvious data errors: one with a price greater than 800,000% of NAV, and another with a price that is 1,000,000% of NAV.
5) Eliminate transactions for which the total amount committed by the seller is less than the unfunded amount.
6) Eliminate transactions for which the total capital committed is less than or equal to zero.
7) Eliminate transactions for which the fund name is missing.
8) Eliminate transactions that do not occur on the most recent transaction date within a calendar quarter for a given fund, where funds are uniquely identified by fund name and vintage.

[26] Individual funds are frequently sold as part of a larger portfolio transaction. In a portfolio transaction, the buyer submits an offer price for an entire portfolio of funds. Prices of the individual funds in the portfolio are then determined subject to the constraint that the size-weighted average of the individual prices equals the winning offer price. On a few rare occasions in our data, however, the individual fund prices are not given, and the price is the same across all funds in the portfolio. In fact, this screen eliminates only 28 transactions.

9) If multiple transactions occur on the most recent transaction date for a given fund/quarter, use only the transaction based on the highest total commitment.
10) If multiple transaction records exist with the same fund name, vintage, and commitment on the most recent transaction date for a given fund/quarter, take the mean transaction price (as a percent of NAV) as the price for this fund/quarter.
11) Eliminate any remaining records for which the same fund name occurs more than once in the same calendar quarter. These are records with the same fund name but different vintages, and are likely indicative of data errors.
12) Of the remaining transactions eliminate any for which the price, as a percent of NAV, is greater than 3 standard deviations away from the mean price across funds for a given quarter.

After cleaning the data as described above, we are left with 3490 fund transactions of which 2258 are for buyout funds and 1484 are for venture funds.

We also classify funds in the cash-flow as buyout or venture based on the fund type as specified by Preqin . Specifically, we classify funds as buyout funds that are labeled "Buyout." We classify funds as venture funds that are labeled "Venture (General)," "Venture Debt," "Growth," "Early Stage," "Early Stage: Seed," "Early Stage: Start-up," and "Expansion / Late Stage."

To merge the transaction and cash-flow data, we first identify funds with identical fund names in the two databases and designate these as a match. Doing so gives us 676 matching funds. We then identify fund names in the transaction (cash-flow) data that are "similar" to fund names in the cash-flow (transaction) data and that also have the same vintage. Fund names A and B are considered similar if fund name A contains the first 10 characters of fund name B anywhere in the fund name string. We then hand check this list to determine which funds match. For example, "New Enterprise Associates 10, L.P." in the transactions data and "New Enterprise Associates X" in the Preqin data are designated as a match. The hand-matching exercise enables us to identify an additional 185 matches across the two databases, so that we have a total of 861 matching funds. Of these, 500 match in the same quarter. For the merged sample, we require that each record have a NAV, PME, and size, which eliminates only a few records. The 500 matching funds account for 1178 transactions unique by fund and quarter.

Figure 1. Transactions per Period, Merged Sample

This figure illustrates the number of transactions we observe per period in the merged sample, as documented in Table 1. In total, the merged sample contains 794 buyout funds and 452 venture funds.

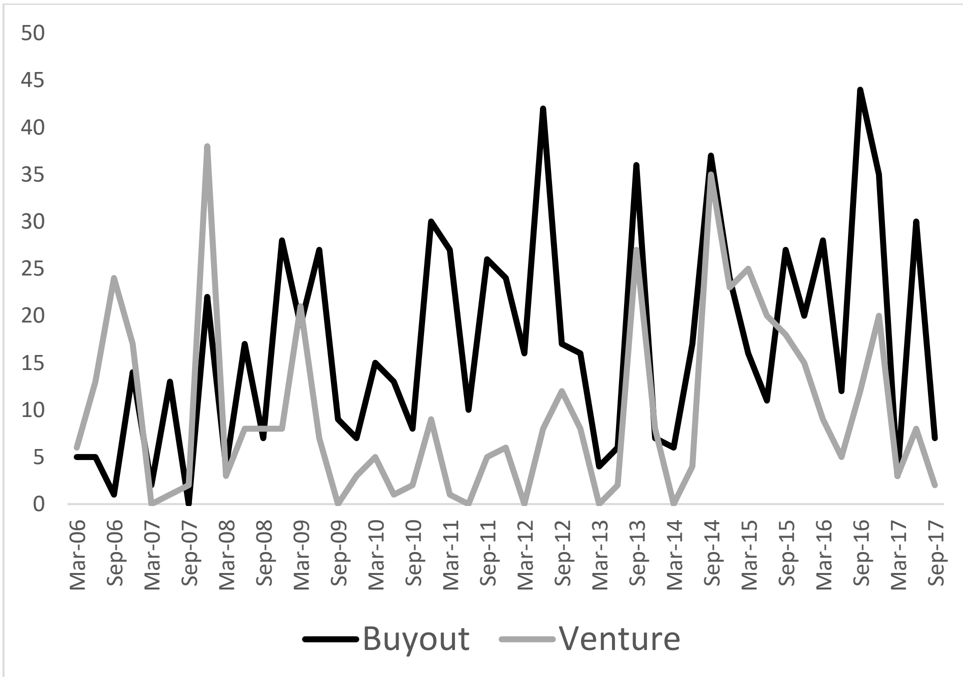
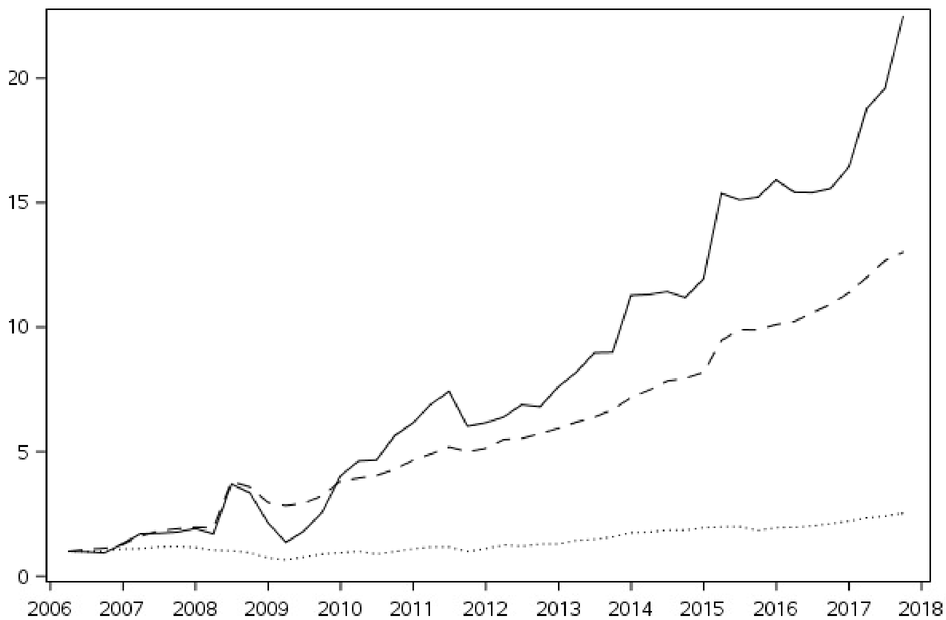


Figure 2. Buyout and Venture Indices Over Time

This figure illustrates the value of investing \$1 in an index at the beginning of 2006. In each panel the solid line represents our size-weighted hedonic transaction-based index, the dashed line represents a size-weighted Preqin NAV-based index, and the dotted line represents the equity market as reported on Ken French's website. The index return each period is calculated as $(V_{t+1} - C_t + D_t)/V_t$ where V_t represents either market value (solid line) or NAV (dashed line) at time t, and C_t, D_t represent calls and distributions made from $t-1$ through t.

Panel A. Buyout



Panel B. Venture

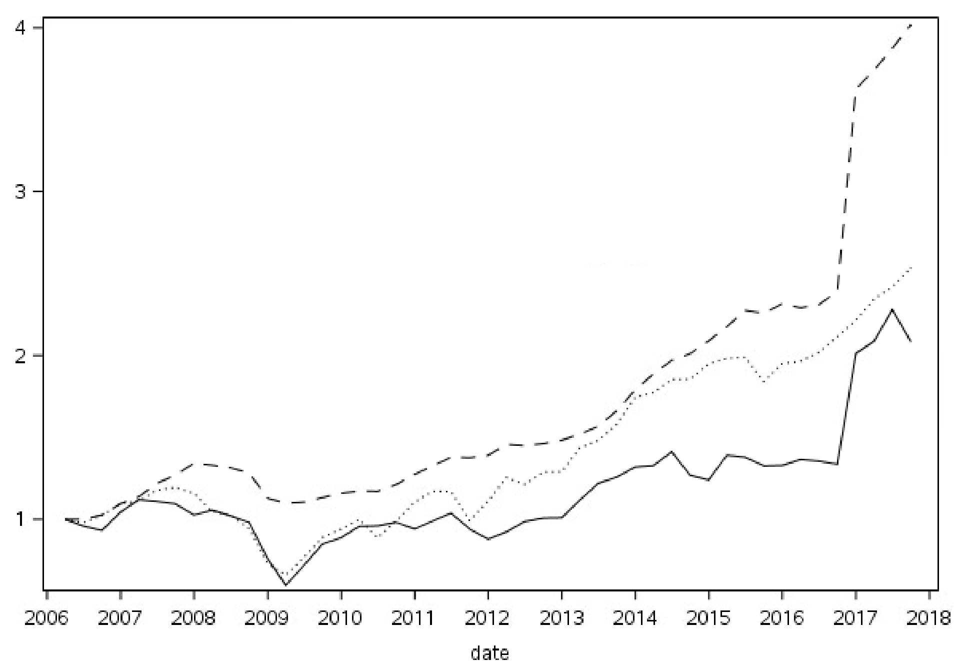


Table 1. Summary Statistics

This table reports summary statistics for the data samples we use. Panel A is for buyout funds while Panel B is for Venture funds. The "transactions sample" is the cleaned sample of all transactions as described in the text. The "merged sample" represents the set of records with matching fund/quarter observations in both the transaction sample and the Preqin Universe. The "compliment sample" contains records in the Preqin Universe with no transactions in the transactions sample. The "fairway merged sample" is the set of records in the merged sample for funds that are 4 to 9 years old, while the "fairway compliment sample" is the set of records in Preqin that are 4 to 9 years old with no transactions in the transactions sample. $\pi_{i,t}$ is the fund price as a fraction of NAV. $Size(\$MM)$ represents total commitments in \$US millions. Age is fund age in years. $Trans\ per\ Qtr$ is the number of transactions per quarter. PME is the Kaplan Schoar (2005) PME using NAV as terminal value for funds that have not yet liquidated. N is the total number of observations in the sample. "Mean" is the average across funds and across time, "Q1" is the 25th percentile, and "Q3" is the 75th percentile.

Panel A. Buyout						
			Preqin Universe		*Prequin 4-9 Years Old*	
		Transactions Sample	*Merged Sample*	*Compliment Sample*	*Fairway Merged*	*Fairway Compliment*
$\pi_{i,t}$	Mean	0.83	0.82		0.89	
	Q1	0.70	0.68		0.80	
	Q3	1.00	1.00		1.03	
Size ($MM)	Mean	4447.0	4461.0	1587.4	5235.4	1844.4
	Q1	1200.0	1200.0	320.7	1700.0	400.0
	Q3	5754.1	5754.1	1750.0	7279.2	2065.0
Age (years)	Mean	9.4	8.30	8.2	6.52	6.39
	Q1	6.0	5.00	3.0	5.00	5.00
	Q3	12.0	11.00	12.0	8.00	8.00
Trans per Qtr	Mean	44.0	17.3		8.7	
	Q1	14.0	7.0		4.0	
	Q3	56.0	27.0		12.0	
PME	Mean	1.19	1.17	1.12	1.16	1.17
	Q1	0.94	0.93	0.87	0.94	0.94
	Q3	1.41	1.35	1.30	1.35	1.35
N		2,066	794	27,761	390	10,060

Table 1. (*continued*)

Panel B. Venture						
			Prequin Universe		*Prequin 4-9 Years Old*	
		Transactions Sample	*Merged Sample*	*Compliment Sample*	*Fairway Merged*	*Fairway Compliment*
$\pi_{i,t}$	Mean	0.80	0.83		0.92	
	Q1	0.60	0.62		0.75	
	Q3	0.99	1.01		1.21	
Size ($MM)	Mean	629.7	632.6	370.8	697.2	403.6
	Q1	283.0	289.5	137.0	375.0	162.8
	Q3	830.0	830.0	450.0	855.7	500.0
Age (years)	Mean	10.8	10.25	9.0	6.73	6.55
	Q1	7.0	7.00	4.0	6.00	5.00
	Q3	14.0	14.00	13.0	8.00	8.00
Trans per Qtr	Mean	31.1	11.0		4.9	
	Q1	9.0	7		2	
	Q3	51.0	14		6	
PME	Mean	0.97	1	0.96	1	1
	Q1	0.56	0.61	0.64	0.66	0.61
	Q3	1	1.16	1	1.14	1.18
N		1,338	452	27,572	165	10,051

Table 2. Explanatory Variable Descriptions

This table describes the explanatory variables used in our hedonic models. The first six variables are "state variables" that are the same across all funds and vary only across time. The last six variables are "fund specific variables" that vary across funds and also (potentially) across time.

State Variables	MB_t	The average end-of-month market-to-book ratio over quarter *t*, calculated using all stocks with share code 10 or 11 on CRSP.
	VOL_t	The annualized standard deviation of the value-weighted portfolio of all stocks in CRSP with share code 10 or 11 using daily data over quarter t.
	$VALUE_t$	The average end-of-month value of the Valuation Confidence Index over quarter *t* from the International Center for Finance at Yale. Insitutional Investors are asked to report their assument of stock market value realtive to fundamental value. The Valuation Confindence Index is the percentage of respondants who think that the market is *not* overvalued relative to fundamentals. We scale this variable by 100.
	$CRASH_t$	The average end-of-month value of the Crash Confidence Index over quarter *t* from the International Center for Finance at Yale. Insitutional Investors are asked to report the probability of a catostrophic market crash in the next six months. The Crash Confindece Index is the percentage of respondants who think that the probability is less than 10%. We scale this variable by 100.
	$MNAV_t$	The average log NAV reported at the end of quarter *t* across funds.
	$MPME_t$	The average Kaplan Schoar (2005) PME at the end of quarter *t* across funds.
Fund SpecificVriables	$LSIZE_i$	The log size of fund *i* (total commitments of all limited partners).
	$NAV_{i,t}$	The log NAV for fund *i* reported as of the end of quarter *t*.
	$PME_{i,t}$	The Kaplan Schoar (2005) PME of fund *i* as of the end of quarter t, calculated using data from Prequin.
	$AGE_{1,t}$	Equals 1 if the year of quarter *t* minus the vintage year is less than 4.
	$AGE_{2,t}$	Equals 1 if the year of quarter *t* minus the vintage year is greater than or equal to 4 and less than or equal to 9.
	$PENSION_i$	The fraction of fund *i* limited partners that are pension funds.

Table 3. Sample Selection Model Parameters

This table reports the estimates of the Heckman (1979) sample selection model of equation (17). Panel A reports estimates of $\boldsymbol{\theta}$ for the "pricing equation", while Panel B reports estimates of $\boldsymbol{\gamma}$ for the selection equation. "Heckman" refers to the full model, while "OLS" refers to the model estaimted by simple OLS with no selection equation. Variables are described in Table 2. Significance at the 1%, 5%, and 10% levels is indicated, respectively, by "***", "**", and "*".

Panel A: Pricing Equation

		Buyout				Venture			
		Heckman		OLS		Heckman		OLS	
		estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
	Intercept	-13.43	-(7.8)***	-13.52	-(8.0)***	6.37	(2.0)**	2.74	(1.0)
State Variables	MB_t	0.24	(3.1)***	0.24	(3.2)***	-0.44	-(1.8)*	0.27	(2.4)**
	VOL_t	0.52	(3.8)***	0.52	(3.8)***	0.81	(2.3)**	0.34	(1.1)
	$VALUE_t$	-0.31	-(2.1)**	-0.30	-(2.1)**	-1.16	-(2.6)**	0.00	(0.0)
	$CRASH_t$	0.70	(2.9)***	0.68	(3.0)***	2.33	(4.2)***	0.98	(2.4)**
	$MNAV_t$	0.93	(8.5)***	0.93	(8.6)***	-0.06	-(0.3)	-0.20	-(1.2)
	$MPME_t$	-0.62	-(1.8)*	-0.60	-(1.8)*	-3.25	-(3.4)***	-0.64	-(1.0)
Fund Specific	$LSIZE_i$	0.00	(0.2)	0.01	(0.8)	-0.12	-(2.6)***	0.01	(0.6)
	$NAV_{i,t}$	0.03	(3.9)***	0.03	(4.1)***	0.04	(2.9)***	0.04	(2.9)***
	$PME_{i,t}$	0.09	(3.4)***	0.09	(3.4)***	0.03	(0.5)	0.02	(0.4)
	$AGE1_{i,t}$	0.03	(0.7)	0.02	(0.6)	0.30	(2.4)**	-0.02	-(0.3)
	$AGE2_{i,t}$	0.10	(3.4)***	0.10	(3.6)***	0.14	(2.1)**	-0.01	-(0.2)
	R-square	33%		33%		18%		21%	

Table 3. (*continued*)

		Panel B: Selection Equation			
		Buyout		Venture	
		estimate	(t-stat)	estimate	(t-stat)
	Intercept	-7.16	-(2.0)**	-11.63	-(2.6)***
State Variables	MB_t	0.26	(2.3)**	2.23	(10.3)***
	VOL_t	-0.10	-(0.3)	-1.22	-(2.9)***
	$VALUE_t$	0.63	(2.8)***	3.88	(8.8)***
	$CRASH_t$	-1.67	-(3.8)***	-4.21	-(7.2)***
	$MNAV_t$	-0.03	-(0.2)	-0.35	-(1.3)
	$MPME_t$	2.15	(2.9)***	7.64	(7.6)***
Fund Specific	$LSIZE_i$	0.42	(26.4)***	0.40	(16.7)***
	$NAV_{i,t}$	0.04	(2.4)**	-0.01	-(0.6)
	$PME_{i,t}$	-0.08	-(1.9)*	0.00	(0.1)
	$AGE1_{i,t}$	-0.91	-(16.1)***	-0.99	-(12.5)***
	$AGE2_{i,t}$	-0.38	-(7.9)***	-0.49	-(9.1)***
	$PENSION_i$	-0.38	-(4.8)***	-0.17	-(2.2)**
	ρ	-0.04	-(0.3)	-0.80	-(7.0)***
	H0: $\rho = 0$	$\chi^2(1)$	(p-val)	$\chi^2(1)$	(p-val)
	Wald	0.10	(0.76)	48.59	(0.00)***
	L.R.	0.02	(0.89)	4.52	(0.03)**
	L.M.	0.00	(0.95)	0.04	(0.83)

Table 4. Buyout Indices 2006-2017

This table reports moment estimates for buyout indices using data from 2006-2017. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices, and Panel C is for the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the merged fairway samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation, $\rho(r_t^O, r_{t-1}^O)$. For the naïve index, β and α are also bias adjusted. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices

	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted		Hedonic Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.37	(2.84)***	0.19	(1.98)**	0.28	(2.50)**	0.39	(2.74)***
β	1.77	(1.90)*	2.10	(3.87)***	1.47	(2.13)**	1.85	(2.27)**
α	0.21	(1.15)	0.00	(0.03)	0.14	(1.31)	0.22	(1.37)
σ	0.52	(1.84)*	0.82	(4.60)***	0.41	(6.01)***	0.69	(3.24)***
Sharpe	0.71	(1.46)	0.23	(1.75)*	0.66	(2.49)**	0.56	(2.38)**
$\rho(r_t, r_{m,t})$	0.55	(2.91)***	0.41	(3.53)***	0.24	(1.32)	0.44	(2.66)***
$\rho(r_t^O, r_{t-1}^O)$	0.02	(0.13)	0.29	(2.13)**	-0.19	-(1.08)	0.12	(0.86)

Panel B: Preqin NAV Based Indices

	Size Weighted		NAV Weighted		NAV Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.26	(3.45)***	0.13	(4.15)***	0.13	(4.19)***	0.28	(3.05)***
β	0.30	(1.69)*	0.35	(3.65)***	0.34	(3.52)***	0.20	(1.23)
α	0.22	(2.51)**	0.09	(3.44)***	0.09	(3.47)***	0.25	(2.52)**
σ	0.30	(2.55)**	0.08	(3.86)***	0.08	(4.04)***	0.31	(3.44)***
Sharpe	0.84	(4.17)***	1.52	(2.08)**	1.51	(2.14)**	0.86	(7.43)***
$\rho(r_t, r_{m,t})$	0.17	(1.25)	0.72	(7.54)***	0.69	(6.41)***	0.12	(1.22)
$\rho(r_t^O, r_{t-1}^O)$	-0.08	-(0.77)	0.54	(3.77)***	0.48	(3.04)***	0.07	(0.52)

Panel C: Difference

	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.11	(1.30)	0.06	(0.82)	0.15	(1.60)	0.11	(1.33)
β	1.47	(1.82)*	1.75	(3.53)***	1.12	(1.79)*	1.65	(2.29) **
α	-0.01	-(0.14)	-0.08	-(1.42)	0.05	(0.56)	-0.03	-(0.29)
σ	0.22	(0.61)	0.74	(4.31)***	0.33	(5.44)***	0.38	(1.68) *
Sharpe	-0.13	-(0.24)	-1.29	-(2.05)**	-0.85	-(1.69)*	-0.30	-(1.52)
$\rho(r_t, r_{m,t})$	0.38	-(0.91)	-0.32	-(6.11)***	-0.46	-(6.56)***	0.32	-(1.08)
$\rho(r_t^O, r_{t-1}^O)$	0.10	(0.84)	-0.24	-(1.37)	-0.67	-(3.72)***	0.06	(0.70)

Table 5. Buyout Indices 2006-2017 Excluding the Financial Crisis

This table reports moment estimates for buyout indices using data from 2006-2017 excluding the years 2008 and 2009. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices, and Panel C is for the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the merged fairway samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1 except for the autocorrelation, $\rho(r_t^O, r_{t-1}^O)$. For the naïve index, β and α are also bias adjusted. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices

	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.28	(4.10)***	0.22	(4.24)***	0.27	(3.98)***	0.34	(2.74)***
β	0.87	(2.58)**	0.72	(2.35)**	0.49	(1.10)	0.75	(2.03)**
α	0.17	(2.24)**	0.13	(2.07)**	0.20	(1.99)**	0.25	(1.98)**
σ	0.36	(0.93)	0.18	(6.28)***	0.30	(5.63)***	0.70	(3.60)***
Sharpe	0.77	(0.92)	1.23	(4.19)***	0.87	(3.54)***	0.49	(3.68)***
$\rho(r_t, r_{m,t})$	0.31	(0.92)	0.43	(3.36)***	0.07	(0.54)	0.16	(2.44)**
$\rho(r_t^O, r_{t-1}^O)$	0.04	(0.30)	-0.09	-(1.14)	-0.47	-(7.38)***	0.18	(1.76)*

Panel B: Preqin NAV Based Indices

	Size Weighted		NAV Weighted		NAV Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.21	(5.30)***	0.16	(8.51)***	0.16	(8.63)***	0.26	(2.73)***
β	0.23	(3.65)***	0.21	(5.66)***	0.20	(5.27)***	0.04	(0.23)
α	0.17	(4.15)***	0.13	(8.70)***	0.13	(8.67)***	0.24	(2.13)**
σ	0.10	(3.82)***	0.05	(5.56)***	0.05	(5.50)***	0.26	(2.32)**
Sharpe	2.00	(7.27)***	3.26	(6.49)***	3.27	(6.40)***	0.96	(10.15)***
$\rho(r_t, r_{m,t})$	0.30	(2.36)**	0.56	(5.03)***	0.54	(4.73)***	0.05	(0.67)
$\rho(r_t^O, r_{t-1}^O)$	0.42	(2.54)**	0.32	(2.44)**	0.28	(2.14)**	0.23	(3.64)***

Panel C: Difference

	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.07	(1.52)	0.06	(1.33)	0.11	(1.72)*	0.08	(1.70)*
β	0.65	(2.05)**	0.52	(1.83)*	0.30	(0.65)	0.71	(1.77)*
α	0.00	(0.00)	0.00	(0.08)	0.07	(0.76)	0.01	(0.11)
σ	0.26	(0.71)	0.13	(5.76)***	0.25	(5.28)***	0.44	(3.89)***
Sharpe	-1.23	-(1.83)*	-2.02	-(6.24)***	-2.40	-(4.80)***	-0.47	-(3.68)***
$\rho(r_t, r_{m,t})$	0.01	-(1.07)	-0.13	-(6.14)***	-0.48	-(4.18)***	0.11	-(0.66)
$\rho(r_t^O, r_{t-1}^O)$	-0.38	-(2.19)**	-0.42	-(2.72)***	-0.75	-(4.37)***	-0.05	-(0.30)

Table 6. Venture Indices 2006-2017

This table reports moment estimates for venture indices using data from 2006-2017. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices, and Panel C is for the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the merged fairway samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1, except for the autocorrelation, $\rho(r_t^O, r_{t-1}^O)$. For the naïve index, β and α are also bias adjusted. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices								
	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.09	(1.29)	0.03	(0.67)	0.08	(0.57)	0.11	(1.68)*
β	1.23	(3.81)***	1.10	(3.77)***	1.43	(3.47)***	1.02	(2.49)**
α	-0.03	-(0.56)	-0.07	-(1.79)*	-0.02	-(0.16)	0.02	(0.28)
σ	0.52	(2.42)**	0.72	(4.19)***	0.59	(4.76)***	0.39	(1.40)
Sharpe	0.16	(0.89)	0.04	(0.56)	0.18	(0.69)	0.28	(0.83)
$\rho(r_t, r_{m,t})$	0.37	(2.85)***	0.24	(3.04)***	0.22	(2.23)**	0.41	(1.88)*
$\rho(r_t^O, r_{t-1}^O)$	0.11	(0.98)	0.25	(2.02)**	-0.41	-(2.25)**	0.05	(0.46)
Panel B: Preqin NAV Based Indices								
	Size Weighted		NAV Weighted		NAV Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.13	(2.64)***	0.08	(3.24)***	0.09	(3.40)***	0.15	(2.80)***
β	0.31	(3.01)***	0.26	(3.30)***	0.26	(3.18)***	0.13	(0.61)
α	0.10	(2.24)**	0.05	(2.42)**	0.06	(2.60)***	0.13	(2.21)**
σ	0.16	(2.60)***	0.06	(4.88)***	0.07	(5.24)***	0.18	(2.90)***
Sharpe	0.80	(4.02)***	1.19	(2.01)**	1.26	(2.13)**	0.81	(4.60)***
$\rho(r_t, r_{m,t})$	0.31	(2.21)**	0.63	(5.03)***	0.61	(4.50)***	0.11	(0.58)
$\rho(r_t^O, r_{t-1}^O)$	0.11	(1.06)	0.52	(4.38)***	0.50	(4.20)***	0.03	(0.46)
Panel C: Difference								
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	-0.05	-(1.13)	-0.05	-(1.31)	-0.01	-(0.11)	-0.04	-(0.98)
β	0.92	(3.33)***	0.84	(3.24)***	1.17	(3.14)***	0.89	(3.23) ***
α	-0.13	-(3.12)***	-0.12	-(3.29)***	-0.08	-(0.62)	-0.12	-(3.01) ***
σ	0.36	(1.44)	0.66	(3.90)***	0.53	(4.21)***	0.21	(0.68)
Sharpe	-0.64	-(3.11)***	-1.15	-(2.11)**	-1.08	-(1.94)*	-0.53	-(1.79) *
$\rho(r_t, r_{m,t})$	0.06	-(1.35)	-0.39	-(5.04)***	-0.39	-(6.24)***	0.30	-(0.37)
$\rho(r_t^O, r_{t-1}^O)$	0.01	(0.11)	-0.27	-(1.84)*	-0.91	-(5.08)***	0.02	(0.28)

Table 7. Venture Indices 2006-2017 Excluding the Financial Crisis

This table reports moment estimates for venture indices using data from 2006-2017 excluding the years 2008 and 2009. Panel A is for our transactions-based indices, Panel B is for Preqin NAV-based indices, and Panel C is for the difference. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the merged fairway samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1, except for the autocorrelation, $\rho(r_t^O, r_{t-1}^O)$. For the naïve index, β and α are also bias adjusted. We currently do not bias correct σ for the naive index or other parameters that depend on σ for the naïve index since doing so would result in negative σ. The moments of the Preqin NAV-based indices are also not bias adjusted since NAV-based indices are not characterized by the types of measurement error discussed in section 3.1.

Panel A: Transactions-Based Indices

	Hedonic Size Weighted		Hedonic Price Weighted		Naïve Price Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.11	(1.82)*	0.05	(1.48)	0.14	(1.03)	0.14	(2.26)**
β	0.68	(2.62)***	0.52	(2.57)**	0.45	(0.56)	0.25	(0.86)
α	0.02	(0.46)	-0.02	-(0.67)	0.12	(0.64)	0.11	(1.71)*
σ	0.20	(3.41)***	0.19	(0.34)	0.59	(4.07)***	0.20	(3.22)***
Sharpe	0.55	(2.76)***	0.22	(0.34)	0.31	(1.07)	0.67	(4.15)***
$\rho(r_t, r_{m,t})$	0.20	(2.19)**	0.33	(0.36)	0.09	(0.90)	-0.04	-(0.26)
$\rho(r_t^O, r_{t-1}^O)$	-0.04	-(0.77)	0.01	(0.12)	-0.33	-(2.43)**	-0.10	-(2.43)**

Panel B: Preqin NAV Based Indices

	Size Weighted		NAV Weighted		NAV Weighted		Equally Weighted	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	0.18	(3.39)***	0.12	(6.39)***	0.12	(6.35)***	0.19	(3.43)***
β	0.23	(3.12)***	0.16	(4.15)***	0.16	(3.88)***	-0.16	-(0.61)
α	0.14	(3.16)***	0.09	(5.80)***	0.10	(5.59)***	0.21	(2.95)***
σ	0.16	(2.36)**	0.05	(11.23)***	0.05	(8.73)***	0.19	(2.70)***
Sharpe	1.05	(5.89)***	2.40	(5.90)***	2.37	(4.54)***	1.03	(6.81)***
$\rho(r_t, r_{m,t})$	0.17	(2.99)***	0.42	(4.85)***	0.38	(5.18)***	-0.11	-(0.57)
$\rho(r_t^O, r_{t-1}^O)$	0.01	(0.15)	0.32	(2.71)***	0.28	(2.30)**	-0.05	-(0.89)

Panel C: Difference

	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
E[r]	-0.07	-(2.03)**	-0.07	-(2.37)**	0.02	(0.14)	-0.06	-(1.87)*
β	0.46	(1.91)*	0.37	(1.82)*	0.30	(0.37)	0.41	(1.95)*
α	-0.12	-(2.86)***	-0.11	-(3.02)***	0.03	(0.14)	-0.11	-(2.77)***
σ	0.03	(2.27)**	0.15	(0.26)	0.54	(3.70)***	0.02	(1.77)*
Sharpe	-0.50	-(1.62)	-2.17	-(3.27)***	-2.07	-(3.59)***	-0.36	-(1.60)
$\rho(r_t, r_{m,t})$	0.03	-(2.09)**	-0.09	-(0.46)	-0.29	-(3.40)***	0.07	(1.23)
$\rho(r_t^O, r_{t-1}^O)$	-0.04	-(0.92)	-0.31	-(2.11)**	-0.61	-(3.59)***	-0.05	-(1.35)

Table 8. Burgiss Index

This table compares moments of the size-weighted hedonic index and the Preqin NAV-based index with the Burgiss index. We create the hedonic indices by applying the coefficients of the pricing models reported in Table 3 to the merged fairway samples as reported in Table 1. Moments of the hedonic indices in Panel A are bias adjusted as discussed in section 3.1, except for the autocorrelation, $\rho(r_t^O, r_{t-1}^O)$. The NAV-based index is created by using NAV as a measure of market value. The moments of the Preqin NAV-based index and the Burgiss index are also not bias adjusted since these indices are not subject by the types of measurement error discussed in section 3.1.

Panel A. Buyout										
	Transactions		NAV		Burgiss		Burgiss - Trans		Burgiss - NAV	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)				
E[r]	0.37	(2.84)***	0.26	(3.45)***	0.10	(2.77)***	-0.26	-(2.17)**	-0.16	(1.97)**
β	1.77	(1.90)*	0.30	(1.69)*	0.45	(4.30)***	-1.32	-(1.44)	0.15	-(0.37)
α	0.21	(1.15)	0.22	(2.51)**	0.05	(1.82)*	-0.16	-(0.88)	-0.17	(1.49)
σ	0.52	(1.84)*	0.30	(2.55)**	0.09	(3.96)***	-0.42	-(1.50)	-0.20	(1.88)*
Sharpe	0.71	(1.46)	0.84	(4.17)***	0.97	(1.64)	0.26	(0.36)	0.13	-(0.31)
$\rho(r_t, r_{m,t})$	0.55	(2.91)***	0.17	(1.25)	0.77	(10.40)***	0.22	(5.18)***	0.60	-(6.18)***
$\rho(r_t^O, r_{t-1}^O)$	0.02	(0.13)	-0.08	-(0.77)	0.50	(3.67)***	0.48	(2.74)***	0.58	-(3.92)***

Table 9. Efficient Frontier

This table reports portfolio weights from efficient frontier calculations. The first row calculates long/short weights using a base set of assets that include corporate bonds, commodities, real estate, 10-year treasuries, and small, mid, and large cap stocks. The second row of the table reports portfolio weights with the base set of assets and a NAV-based private equity index calculated by Burgis. The third row reports portfolio weights with the base set of assets and the hedonic buyout and venture indexes. Expected annualized returns are calculated in excess of risk free returns. The sample period runs from 2006-2017, the years our hedonic indexes are able to be estimated. Equities data come from Ken French's web site. Small cap equities are the bottom 30th percentile, mid cap equities are the middle 40 percent, and large cap are the largest 30 percent. Estimates also utilize a corporate bond index from Blackrock and Vanguard ETFs for commodities (VAW) and real estate (VNQ). Ten year treasury data are from the St. Louis Fed's FRED database.

	Asset Weights										**Moments**		
Asset Mix	*Corporate Bonds*	*Commodities*	*Real Estate*	*10-Yr Treasury*	*Small Cap*	*Mid Cap*	*Large Cap*	*Burgis Index*	*Hedonic Buyout Index*	*Hedonic Venture Index*	*E[r]*	*Sigma*	*Sharpe*
Base	0.65	-0.41	-0.12	-0.03	-0.58	1.12	0.37	--	--	--	0.08	0.07	1.00
Burgis Included	0.42	-0.38	-0.08	0.00	-0.20	0.58	-0.05	0.72	--	--	0.09	0.05	1.59
Buyout and Venture Included	0.61	-0.41	-0.19	0.00	-0.19	0.26	0.74	--	0.11	0.07	0.10	0.08	1.25

Table 10. Market-to-Book Ratios

This table reports year-end average market-to-book ratios for buyout funds. Market values for each fund are calculated using the following procedure. We begin by assuming that the market value of the fund is equal to NAV in years one through four of the fund's life. We then calculate the market value each quarter from years 5-9 for fund *i* using the following formula:

$$market\ value_{i,t=}\ market\ value_{i,t-1} * (1 + r_t) + Calls_t - Distributions_t\ .$$

For the first quarter in year five, we use NAV as the preceding quarter's market value. The aggregate market-to-book ratio reported in this table is calculated as the sum of the individual fund's market value within each quarter divided by the sum of the individual fund's NAV in each quarter. We report the resultant market-to-book ratio for Q4 of each year, with the exception of 2017, where we report values as of Q2 due to data limitations. Panel A reports results for buyout funds. Panel B reports results for venture funds.

Panel A. Buyout Funds (4-9 Years old)

	Vintage Year											
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
2006	1.14	--	--	--	--	--	--	--	--	--	--	--
2007	1.03	1.02	--	--	--	--	--	--	--	--	--	--
2008	0.77	0.65	0.76	--	--	--	--	--	--	--	--	--
2009	0.97	0.86	1.00	1.57	--	--	--	--	--	--	--	--
2010	1.28	1.03	1.23	1.92	1.05	--	--	--	--	--	--	--
2011	1.28	0.92	1.12	1.80	0.91	0.87	--	--	--	--	--	--
2012	--	0.98	1.25	2.20	0.97	1.01	1.05	--	--	--	--	--
2013	--	--	1.80	3.20	1.19	1.23	1.25	1.13	--	--	--	--
2014	--	--	--	3.95	1.14	1.17	1.17	0.99	1.02	--	--	--
2015	--	--	--	--	1.28	1.28	1.31	1.05	1.02	1.03	--	--
2016	--	--	--	--	--	1.22	1.28	0.91	0.91	0.93	1.00	--
2017	--	--	--	--	--	--	1.40	0.94	0.99	0.98	1.04	1.13